Beginning Algebra
Math 100B

Math Study Center
BYU-Idaho
Preface

This math book has been created by the BYU-Idaho Math Study Center for the college student who needs an introduction to Algebra. This book is the product of many years of implementation of an extremely successful Beginning Algebra program and includes perspectives and tips from experienced instructors and tutors.

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We hope that it will be helpful to you as you take Algebra this semester.

The BYU-Idaho Math Study Center
# Math 100B
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Chapter 1: ARITHMETIC & VARIABLES

Overview

Arithmetic
1.1 Decimals, Positives and Negatives
1.2 Fractions
1.3 Exponents and Roots
1.4 Percents
1.5 Rounding and Estimation, Order of Operations

Variables and Formulas
1.6 Variables and Formulas (Geometry)
1.7 Laws of Simplifying
Numbers are everywhere. The rules they follow help us to know how to govern bank accounts, build buildings, and predict orbits of planets. We use them to measure heights, lengths, and amounts of everything. Mastery of these rules is necessary to understand everything that goes on around us.

We will learn how to handle and feel comfortable in all of these situations.

### EXAMPLES

We will learn how to handle and feel comfortable in all of these situations.

<table>
<thead>
<tr>
<th>Arithmetic</th>
<th>What kind of problem</th>
<th>Where it might be found</th>
</tr>
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<tbody>
<tr>
<td>$30.00 – $22.73</td>
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<td>Finding change at the store</td>
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<td>6% of $18$</td>
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<td>$9.3 \times 10^7$</td>
<td>Scientific Notation</td>
<td>News article reporting on space with the number of miles to the sun</td>
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<tr>
<td>$7 \times 3.59$</td>
<td>Decimal multiplication</td>
<td>The total price of 7 items at $3.59 each</td>
</tr>
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There are two different types of problems in algebra: the “simplify” type and the “solve” type. In this chapter we will look at the simplify type. We will cover solving in chapter 2.

**ALGEBRA: Two Types of Problems**

<table>
<thead>
<tr>
<th>SIMPLIFY (CH.1)</th>
<th>SOLVE (CH.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No “=” signs (or &gt;,&lt;, etc.)</td>
<td>Uses “=” signs (or &gt;,&lt;, etc.)</td>
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<tr>
<td>Example: 2x + 3x</td>
<td>Example: 3x = 15</td>
</tr>
<tr>
<td></td>
<td>Find out what x equals</td>
</tr>
</tbody>
</table>

**SIMPLIFY:** Though every number can be written in many different ways, there is usually one way that is most accepted and used. To change the form of a number to this standard is to simplify.

**Example:** If someone asks your age, and you told them √441, or $\frac{63}{3}$, they would probably look at you oddly, but these are both ways of saying 21. We simplify when we take square roots, or reduce fractions as far as we can.

**LIKE THINGS:** In addition and subtraction we must only deal with like things.

**Example:** If someone asks you what 5 sheep plus 2 sheep is, you would be able to tell them 7 sheep.

5 sheep + 2 sheep = 7 sheep

What if they asked you what 5 sheep plus 2 penguins is? We really can’t add them together, because they aren’t like things. Similarly, we can add 5x plus 2x to get 7x, but we can’t add 5x plus 2y. You will see this more in section 6.

**Important Note:** We do not need like things for multiplication and division.

**NEGATIVE:** The negative sign means “opposite direction,” as seen on the number line below.

Example: −4.5 is just 4.5 in the opposite direction

Example: $-\frac{7}{8}$ is just $\frac{7}{8}$ in the opposite direction.

Example: −7 − 5 = −12, because they are both headed in that direction.
PLACE VALUES: Every place on the left or right of the decimal holds a certain value. To the left of the decimal, the values are ones, tens, hundreds, thousands, and so forth. On the right of the decimal, the place values are tenths, hundredths, thousands, and so forth.

\[ 3,451.972 \]

In this example, there are 3 thousands, 4 hundreds, 5 tens, 1 one, 9 tenths, 7 hundredths, and 2 thousandths.

DECIMAL: Deci- is a prefix meaning 10. Since every place value is either 10 times larger or smaller than the place next to it, we call each place a decimal place.

I. Addition of Decimals

The process for adding decimals is almost identical to adding non-decimal numbers.

**Addition of Decimals**

1. Line up decimals
2. Add in columns
3. Carry by 10’s

**EXAMPLE**

Add \( 3561.5 + 274.38 \)

\[
\begin{align*}
3561.5 & \\
+ & 274.38 \\
\hline
3835.88 & \\
\end{align*}
\]

Answer: 3835.88

Section 1.1
II. **Subtraction of Decimals**

Again, subtracting decimals isn’t much different from subtracting other numbers. Remember that the biggest number goes on top, whether it’s positive or negative. Then at the end, the final answer will have the same sign as the larger “stronger” number.

**Subtraction of Decimals**

1. Biggest on top
2. Line up decimals; subtract in columns
3. Borrow by 10’s
4. Strongest wins

**EXAMPLE**

\[
\begin{align*}
\text{Subtract } & 283.5 - 3476.91 \\
- 3476.91 & \\
283.5 & \quad 1. \text{Biggest on top} \\
- 3 & 476.91 \\
2 & 83 \ 5 \\
3 & 41 \\
- 3 & 4 \ 17 \ 6 \ 9 \ 1 \\
2 & 83 \ 5 \\
3 & 193.41 \\
3 & 34 \ 17 \ 6 \ 9 \ 1 \\
2 & 83 \ 5 \\
- 3 & 193.41
\end{align*}
\]

Answer: -3193.41

III. **Multiplication of Decimals**

Multiplying decimals is not any more difficult that multiplying other numbers. The process is the same except that you must account for the decimals at the end. It’s literally as easy as 1-2-3. When you finish a problem, go back and count how many numbers were past the decimal point. Then count that many places to the left of your final answer and put a decimal there. Also, remember to keep track of whether the answer is positive or negative.
**Multiplication of Decimals**

1. Multiply each place value
2. Carry by 10’s
3. Add
4. Right size: 1. Add up zeros or decimals 2. Negatives

**EXAMPLES**

**3**
An example without decimals:
**Multiply 29742 × 538**

```
29742
x 538
```

- 237936
- 892260
  +14871000
  16001196

Start:
```
7 5 3 1
29742
x 8
237936
```

Next:
```
2 2 1
29742
x 30
892260
```

Last:
```
4 3 2 1
29742
x 500
14871000
```

**Answer:** 16,001,196

**4**
Final example with decimals:
**Multiply -7,414.3 × 9.46**

```
-7414.3
x  9.46
```

- 444858
- 2965720
  +66728700
  -70139278

Start:
```
2 2 1
74143
x 6
444858
```

Next:
```
1 1 1
74143
x 40
2965720
```

Last:
```
3 1 3 2
74143
x 900
66728700
```

**Right size. Total number of decimal places = 3. Answer is negative.**

**Answer:** -70,139,278
IV. Division of Decimals

To divide decimals or do a problem that results in a decimal, follow the usual process but don’t stop when you get to a decimal. Write the decimal point but add zeros after that place and continue dividing as usual.

**Division of Decimals**

1. Set up  
2. Add zeros  
3. Divide into first  
4. Multiply  
5. Subtract  
6. Drop Down  
7. Write answer

**EXAMPLES**

Divide \( 429 \div 8 = ? \)

\[
\begin{array}{c|c}
5 \\
8 \overline{)429} \\
-40 \\
\hline
29 \\
-24 \\
\hline
5
\end{array}
\]

Step 2. We know that 8 goes into 42 about 5 times.

Step 3. Multiply 5x8

Step 4. Subtract.

Step 5. Bring down the 9 to continue on.

Repeat 2-5

Step 2: 8 goes into 29 about 3 times.

Step 3: Multiply 3x8

Step 4: Subtract.

8 doesn’t go into 5 (remainder)

Which means that \( 429 \div 8 = 53 \text{ R } 5 \)

or in other words \( 429 \div 8 = 53 \frac{5}{8} \)
Divide $5875 \div 22 = ?$

Step 2: 22 goes into 58 about 2 times.
Step 3: Multiply $2 \times 22 = 44$

Step 4: Subtract.

Step 5: Bring down the next column

22 goes into 147 about ???? times.

Let’s estimate.

2 goes into 14 about 7 times – try that.
Multiply $22 \times 7 = 154$
Oops, a little too big

Since 7 was a little too big, try 6.
Multiply $6 \times 22 = 132$

Subtract.

Bring down the next column.

22 goes into 155 about ????? times.
Estimate.

2 goes into 15 about 7 times. Try 7
Multiply $22 \times 7 = 154$. It worked.

Subtract.

Remainder 1

Answer: 267 R 1 or $267 \frac{1}{22}$

An example resulting in a decimal:

Write $\frac{4}{9}$ as a decimal:

$9 \overline{)4.0000}$ Step 1: Set it up. Write a few zeros, just to be safe.
Step 2: Divide into first.
9 goes into 40 about 4 times.
Step 3. Multiply $4 \times 9 = 36$

Step 4. Subtract.
Step 5. Bring down the next column.
Repeat steps 2-4
Step 2: 9 goes into 40 about 4 times.
Step 3: Multiply $4 \times 9 = 36$

Step 4: Subtract.
Step 5. Bring down the next column.
Repeat steps 2-4
Step 2: 9 goes into 40 about 4 times.
Step 3: Multiply $4 \times 9 = 36$

This could go on forever!

Thus \( \frac{4}{9} = .44444\ldots \) which we simply write by \( .\bar{4} \)

The bar signifies numbers or patterns that repeat.

---

Two final examples:

\[ 358.4 \div -(0.005) \]

\[ \frac{358.4}{-(0.005)} \]

Step 1. Set it up and move the decimals

\[ 5 \overline{358400} \]

Step 2. Divide into first

\[ 7 \]

\[ 5 \overline{358400} \]

Step 3. Multiply down

\[ 35 \]

\[ 7 \]

\[ 5 \overline{358400} \]

Step 4. Subtract

\[ -35 \]

\[ 08 \]

Step 5. Bring down

\[ 9 \]

\[ 31 \overline{2960.00} \]

\[ 279 \]

\[ 9 \]

\[ 31 \overline{2960.00} \]

\[ -279 \]

\[ 150 \]
One negative in the original problem gives a negative answer.

The decimal obviously keeps going. We will discuss appropriate rounding in a later section.
Two negatives make a positive

- **True in Multiplication and Division** – Since a negative sign simply means other direction, when we switch direction twice, we are headed back the way we started.
  
  Example: \(-(-5) = 5\)
  Example: \(-(-2)(-1)(-3)(-5) = -30\)
  Example: \(-(-40 ÷ -8) = -5\)

- **False in Addition and Subtraction** – With addition and subtraction negatives and positives work against each other in a sort of tug of war. Whichever one is stronger will win.

  **Example:** Debt is negative and income is positive. If there is more debt than income, then the net result is debt. If we are $77 in debt and get income of $66 then we have a net debt of $11
  
  \[-77 + 66 = -11\]

  On the other hand if we have $77 dollars of income and $66 of debt, then the net is a positive $11
  
  \[77 − 66 = 11\]

  **Example:** Falling is negative and rising is positive. An airplane rises 307 feet and then falls 23 feet, then the result it a rise of 284 feet:
  
  \[307 − 23 = 284\]

  If, however, the airplane falls 307 feet and then rises 23 feet, then the result is a fall of 284 feet:
  
  \[-307 + 23 = -284\]

  **Other examples:** Discount is negative and markup or sales tax is positive. Warmer is positive and colder is negative. Whichever is greater will give you the sign of the net result.
### 1.1 EXERCISE SET

**Add or subtract.**

1. \(12 + (-7)\)  
2. \(3 - 11\)  
3. \(-15 - 24\)  
4. \(38 - 93\)  
5. \(186.4 + 57.06\)  
6. \(18 + (-73)\)  
7. \(-3 - (-18)\)  
8. \(15 - (-348)\)  
9. \(2,578 + 389.4\)  
10. \(973 - 3,284\)  
11. \(58.93 - 17.986\)  
12. \(2047.5 - 93.72\)

**Multiply.**

13. \(20 \cdot (-7)\)  
14. \(13 \cdot 4\)  
15. \(-15 \cdot (-24)\)  
16. \(3658 \cdot (15)\)  
17. \(18 \cdot (-73)\)  
18. \((-3)(-18)\)  
19. \(54.8 \times 7284.9\)

**Divide.**

22. \(37 \div 5\)  
23. \(23 \div 4\)  
24. \((-43) \div 6\)  
25. \(\frac{28}{3}\)  
26. \(5,968.4 \div 9\)  
27. \((-56) \div 14\)  
28. \(28.4 \div (-2.1)\)  
29. \(365.8 \div 0.5\)  
30. \(0.07 \div 0.006\)

**Change these fractions into decimals.**

31. \(\frac{3}{8}\)  
32. \(\frac{2}{9}\)  
33. \(\frac{12}{5}\)  
34. \(\frac{3}{4}\)

**Find the final.**

35. Price: $365.29  
   Discount: $79  
   Final:  
36. Temp.: 39° F  
   Warmer: 23° F  
   Final:  
37. Altitude: 2,349 ft  
   Rise: 821 ft  
   Final:  
38. Debt: $2,000  
   Income: $487  
   Final:

**Preparation. Read section 1.2 and do the following:**

39. Add.  
   \(0.3 + 0.6\)  
   \(0.03 + 0.06\)  
   \(0.003 + 0.006\)  
   \(0.0003 + 0.0006\)

40. Add.  
   \(\frac{3}{10} + \frac{6}{10}\)  
   \(\frac{3}{100} + \frac{6}{100}\)  
   \(\frac{3}{1,000} + \frac{6}{1,000}\)  
   \(\frac{3}{10,000} + \frac{6}{10,000}\)
Answers
1. 5
2. $-8$
3. $-39$
4. $-55$
5. 243.46
6. $-55$
7. 15
8. 363
9. 2,967.4
10. $-2,311$
11. 40.944
12. 1953.78
13. $-140$
14. 52
15. 360
16. 54,870
17. $-1314$
18. 54
19. 399,212.52
20. 0.000000012
21. 33,000,000
22. 7.4
23. $5.75$ or $5\frac{3}{4}$
24. $-7.1\overline{6}$ or $-7\frac{1}{6}$
25. $9.\overline{3}$ or $9\frac{1}{3}$
26. 663.1$\overline{5}$
27. $-4$
28. $-13.523...$
29. 731.6 or $731\frac{3}{5}$
30. $11.\overline{6}$ or $11\frac{2}{3}$
31. 0.375
32. 0.2
33. 2.4
34. 0.75
35. $286.29$
36. $62^\circ F$
37. 3,170 ft
38. $-1,513$, or $1513$ in debt
39. In class
40. In class
Decimals are only one way of representing parts of a whole. Fractions are another way, and can often be a more exact way of representing a part. For example, one third in decimals is 0.33333… and it doesn’t stop repeating. But as a fraction, it is easily written as \(\frac{1}{3}\). Other fractions work the same way.

Now that you have learned about decimals, let’s take a look at some fractions. If you notice in the examples below, decimals are really just another way of writing fractions where the denominator of the fraction is always a multiple of ten.

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4 + .3 = .7</td>
<td>(\frac{4}{10} + \frac{3}{10} = \frac{7}{10})</td>
</tr>
<tr>
<td>.21 + .73 = .94</td>
<td>(\frac{21}{100} + \frac{73}{100} = \frac{94}{100})</td>
</tr>
<tr>
<td>3 \times .25 = 75</td>
<td>(3 \times \frac{25}{100} = \frac{75}{100})</td>
</tr>
</tbody>
</table>
NUMERATOR: The top of a fraction. This is always an integer, never a decimal.

DENOMINATOR: The bottom of the fraction. This is always an integer, never a decimal, and never zero.

SIMPLIFY: Fractions are simplified when the numerator and denominator have no factors in common. You can also say that the fraction is reduced.

Example: The fraction \( \frac{15}{30} \) can be simplified or reduced down to \( \frac{1}{2} \).

ONE: Any number divided by itself is 1.


Example: You cannot add \( \frac{2}{3} \) (2 thirds) and \( \frac{1}{6} \) (1 sixth) because they not like things. In order to add them, they must have common denominators.

\[
\frac{2}{3} + \frac{1}{6} = \frac{2 \cdot 6 + 1 \cdot 3}{6} = \frac{12 + 3}{6} = \frac{15}{6}
\]

You can find common denominators in three main ways: 1) Simple observation, as in the above example. You can clearly see that 6 is a possible common denominator. 2) Multiply the two denominators together. In the above example, that would result in a common denominator of 18, which would still give you the right answer, but you would be dealing with larger numbers. 3) Use prime factorization to find the least common denominator.

LEAST COMMON DENOMINATOR (LCD): As the name indicates, this is the lowest possible common denominator between two or more fractions. There are an infinite number of possible common denominators, but usually the easiest one to choose is the lowest or least one.

PRIME FACTORIZATION: Breaking a number into smaller and smaller factors until it cannot be broken down further. Prime factorization is one of the ways to get the least common denominator for fractions that have large denominators. You crack them open and see what they are made of. Scientists use a scalpel or microscope. Mathematicians use prime factorization. Remember that a prime number has no other factors besides itself and 1. The first few prime numbers are 1, 2, 3, 5, 7, and 11, but the list goes on.

Example: Find the prime factorization of 24. Let’s use a factorization tree so you can see the process.

\[
\begin{align*}
24 & \quad \text{Factors of 24 are 6 and 4, because } 6 \times 4 = 24. \\
6 & \quad \text{We can break up 6 and 4 into even smaller factors.} \\
2 & \quad 2 \quad 2 \quad 2 \quad \text{Each of these final factors is prime. We have found the prime factorization: } 2 \times 2 \times 2 \times 3.
\end{align*}
\]
Example: Consider the fractions \(\frac{3}{4}\) and \(\frac{5}{6}\). There are many numbers that could work as common denominators, but we can use prime factorization to find the least common denominator. We will find the prime factorization of each denominator, and compare them to find a common one.

The prime factorization for 4 is \(2 \times 2\), and for 6 it is \(2 \times 3\). For the LCD, we need a number that has each of these prime factors in it. However, we only need two 2’s because 2 is repeated in the prime factorization of each of these numbers. The LCD is the number whose prime factorization is \(2 \times 2 \times 3\), which is 12.

*The number found for the LCD is also called the LCM: Least Common Multiple. The only difference is that the LCM might not be used as the denominator of a fraction.

1. **Addition of Fractions**

Adding fractions is another way of adding “like things.” We have to make all the fractions have a common denominator, and then they can be added.

**Addition of Fractions**

1. Get a common denominator between the fractions
2. Add the numerators
3. Carry over denominator

**EXAMPLES**

1. Add \(\frac{13}{30} + \frac{7}{12}\)

Step 1: Common denominator. If we multiply the denominator here, we’ll have some big numbers to work with. Let’s use prime factorization to find the LCD.

Prime factorization of 30: \(2 \times 3 \times 5\)
Prime factorization of 24: \(2 \times 2 \times 3\)

We need a number whose factors include each of these:

\[2 \times 2 \times 3 \times 5 = 60\]

\[\frac{13 \times 2}{30 \times 2} = \frac{26}{60}\]
\[\frac{7 \times 5}{12 \times 5} = \frac{35}{60}\]

Step 2: Now that the denominators are the same, add the numerators.

\[\frac{26 + 35}{60} = \frac{61}{60}\]

Step 3: Carry the denominator across.

Answer: \(\frac{61}{60}\)
II. Subtraction of Fractions
Once again, fractions need a common denominator in order to be subtracted.

Subtraction of Fractions
1. Biggest on top
2. Common denominator; Sub. numerators
3. Borrow by denominator
4. Strongest wins

EXAMPLES

2 Subtract \( \frac{5}{9} - 3 \frac{1}{3} \)

\[
\begin{align*}
3 \frac{1}{3} &- \frac{5}{9} \\
\frac{3}{3} &- \frac{5}{9} \\
\frac{3}{3} &- \frac{5}{9} \\
\frac{3+9}{9} &- \frac{5}{9} \\
\frac{7}{9} &- \frac{5}{9} \\
\frac{2}{9} &- \frac{2}{9} \\
\end{align*}
\]

3 \frac{1}{3} is bigger, so put it on top.

The common denominator is 9, so change the \( \frac{1}{3} \) to a \( \frac{3}{9} \).

Subtract the numerators. Borrow by denominator as needed.

Answer: \( 2 \frac{7}{9} \)

III. Multiplication of Fractions
When multiplying fractions, common denominators are not needed. This is different from addition and subtraction.

Multiplication of Fractions
1. No common denominators
2. Multiply numerators
3. Multiply denominators
Multiply $\frac{5}{6} \times \frac{1}{3}$

For multiplication don’t worry about getting common denominators

Multiply the numerators straight across

Multiply the denominators straight across

Answer: $\frac{5}{18}$

IV. Division of Fractions

Dividing fractions is an interesting idea, because a fraction itself is a division (i.e. $\frac{1}{2}$ can also be said as 1 divided by 2). Because of this, there is a special process for dividing fractions that actually simplifies it. To divide a number by a fraction, reciprocate the fraction and multiply instead. Now you’re doing a multiplication problem, one you already know how to do.

**Division of Fractions**

1. Change any fractions into improper fractions.
2. Keep the first fraction the same, change the division sign to multiplication, and flip the second fraction’s numerator and denominator: Keep it, change it, flip it.
3. Multiply straight across.
EXAMPLES

4  Divide $\frac{3}{7} \div \frac{2}{3}$

Step 1: Turn the fractions into improper fractions

$\frac{3 \times 7}{7} + \frac{4}{7} \div \frac{2}{3}$

Step 2: Keep the first fraction the same
Change the division sign to a multiplication sign
Flip the second fraction’s numerator and denominator

$\frac{25}{7} \div \frac{2}{3}$

Step 3: Multiply straight across the numerator and denominator

$\frac{25}{7} \times \frac{3}{2} = \frac{75}{14}$

Answer: $\frac{75}{14}$

5  Divide $\frac{2}{5} \div 1 \frac{3}{4}$

Turn the fractions into improper fractions

$\frac{2}{5} \div \frac{1 \times 4}{4} + \frac{3}{4}$

Keep the first fraction the same
Change the division sign to a multiplication sign
Flip the second fraction’s numerator and denominator

$\frac{2}{5} \div \frac{7}{4}$

Multiply straight across the numerator and denominator

$\frac{2 \times 4}{5 \times 7} = \frac{8}{35}$

Answer: $\frac{8}{35}$
Perform the indicated operation.

1. \(16 - 5\)  
2. \(18.63 + 5\)  
3. \(48.85 - 27.3\)  
4. \(2.6 \times -13\)  
5. \(38 \div .02\)  
6. \(-18.36 \div 3\)

Find the prime factorization.

a. 60  
b. 630  
c. 225  
d. 210

Find the lowest common multiple (LCM).

e. 35 & 21  
f. 27 & 39  
g. 108 & 32  
h. 1500 & 180

Add.

7. \(\frac{1}{6} + \frac{1}{3}\)  
8. \(8 + \frac{2}{3}\)  
9. \(-\frac{2}{9} + 5\)

10. \(-\frac{4}{9} + \frac{1}{8}\)  
11. \(-\frac{5}{6} + \frac{3}{7}\)  
15. \(\frac{7}{8} + \frac{3}{5}\)

Subtract.

16. \(\frac{5}{8} - \frac{1}{2}\)  
19. \(-\frac{1}{6} - \frac{2}{9}\)  
20. \(\frac{1}{8} - \frac{5}{6}\)

22. \(4\frac{3}{7} - 1\frac{2}{3}\)  
23. \(\frac{7}{8} - 2\frac{5}{9}\)  
24. \(1\frac{2}{3} - 6\frac{7}{8}\)

Multiply.

25. \(3 \times \frac{1}{12}\)  
26. \(\frac{4}{5} \times \frac{1}{6}\)  
27. \(-\frac{5}{8} \times \frac{1}{3}\)

28. \(\frac{4}{5} \times \frac{7}{12}\)  
29. \(\frac{9}{13} \times \frac{2}{5}\)  
30. \(-\frac{6}{7} \times -\frac{2}{9}\)

Divide.

34. \(\frac{5}{12} \div \frac{1}{3}\)  
35. \(\frac{6}{9} \div 6\)  
36. \(-\frac{5}{4} \div -\frac{3}{8}\)

37. \(\frac{2}{5} \div \frac{9}{10}\)  
38. \(-\frac{3}{4} \div \frac{1}{4}\)  
39. \(\frac{7}{11} \div \frac{5}{6}\)

40. \(-\frac{3}{4} \div \frac{7}{8}\)  
41. \(\frac{7}{40} \div -\frac{3}{10}\)  
42. \(-\frac{6}{44} \div -\frac{1}{4}\)

Preparation.

After reading some of section 1.3, find the following:

43. \(2^5 = \)  
44. \(3.26 \times 10^{11} = \)
Answers

1. 11
2. 23.63
3. 21.55
4. −33.8
5. 1900
6. −6.12
a. \(2 \times 2 \times 3 \times 5\)
b. \(2 \times 3 \times 3 \times 5 \times 7\)
c. \(3 \times 3 \times 5 \times 5\)
d. \(2 \times 3 \times 5 \times 7\)
e. 105
f. 351
g. 864
h. 4500
7. \(\frac{1}{2}\)
8. \(\frac{26}{3}\)
9. \(\frac{43}{9}\)
10. \(-\frac{23}{72}\)
11. \(-\frac{53}{42}\)
12. \(\frac{59}{40}\)
13. \(\frac{1}{8}\)
14. \(-\frac{7}{18}\)
15. In Class.
16. In Class.
17. \(-\frac{17}{24}\)
18. \(\frac{16}{21}\)
19. \(-\frac{49}{72}\)
20. \(-\frac{5}{24}\)
21. \(\frac{1}{4}\)
22. \(\frac{2}{15}\)
23. \(-\frac{5}{24}\)
24. \(\frac{7}{15}\)
25. \(-\frac{18}{65}\)
26. \(\frac{4}{21}\)
27. \(\frac{5}{4}\)
28. \(\frac{1}{9}\)
29. \(\frac{10}{3}\)
30. \(\frac{4}{9}\)
31. \(-3\)
32. \(\frac{42}{55}\)
33. \(-\frac{6}{7}\)
34. \(-\frac{7}{12}\)
35. \(\frac{6}{11}\)
36. In Class.
Simplification is a useful principle; it helps us write things in smaller or shorter ways without changing the value. We used fractions sometimes in order to represent parts in a simpler way than a decimal (remember \( \frac{1}{3} \)?). Sometimes it gets to be a hassle to write long multiplication problems, just as it was to write long addition problems. In the scientific community especially, using decimals is the quickest way, but the numbers can get to be a hassle to write out – who wants to put all the 0’s behind 3 billion each time you write it?

### LAWS & PROCESSES

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### EXAMPLE

<table>
<thead>
<tr>
<th>Long Way</th>
<th>Short Way</th>
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<tbody>
<tr>
<td>( 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 )</td>
<td>( 3^8 )</td>
</tr>
<tr>
<td>( 3,490,000,000,000,000 )</td>
<td>( 3.49 \times 10^{15} )</td>
</tr>
<tr>
<td>( 0.0000000000251 )</td>
<td>( 2.51 \times 10^{-11} )</td>
</tr>
</tbody>
</table>
What does this mean? Is it the same as $2 \times 4$? Does it equal 8?

When we see a little number next to a big number, it looks difficult, challenging, but it’s not…it’s just a **shortcut**.

The little number next to the big number is called an **exponent**.

(other names for exponents are **power** or **degree**)

$2^4$ means $2 \times 2 \times 2 \times 2 \rightarrow$ four two’s all being multiplied

Say the following out loud…

“$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$”

How many three’s are there? Now say…

“$3^{10}$” or “three to the tenth”

Which sounds easier?

Hence, **exponents** are **shortcuts**!

**Exponent/Power**: The shorthand to explain how many times something is multiplied by itself.

**Example**: $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$ is the same as $2^7 = 128$

**Root**: Like working an exponent backwards, roots find out what number or variable was raised to a power.

**Example**: We know that $2^7 = 128$, so to work it backwards, we say:

$2 = \sqrt[7]{128}$ or if 128 was cut into 7 equal parts multiplied together, then those parts would all be 2’s.

We can use these “shortcuts” with variables or numbers, for example:

$x \cdot x \cdot x \cdot x \cdot x = x^5$ just as $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

How do we say it?

$x^5$ = “$x$ to the fifth” $2^4$ = “two to the fourth”
The Three E’s of Exponents

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<th>Exponential Notation</th>
<th>Expanded Notation</th>
<th>Evaluated Notation</th>
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<tbody>
<tr>
<td>(2^1)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(2^2)</td>
<td>(2 \times 2)</td>
<td>4</td>
</tr>
<tr>
<td>(2^3)</td>
<td>(2 \times 2 \times 2)</td>
<td>8</td>
</tr>
<tr>
<td>(2^4)</td>
<td>(2 \times 2 \times 2 \times 2)</td>
<td>16</td>
</tr>
<tr>
<td>(2^4)</td>
<td>(2 \times 2 \times 2 \times 2 \times 2)</td>
<td>32</td>
</tr>
</tbody>
</table>

Using the 3 E’s of Exponents...

a) Write \(4^3\) in expanded form.  
Answer: \(4 \times 4 \times 4\)

b) Write \(2 \times 2 \times 2 \times 2\) in exponential form.  
Answer: \(2^5\)

c) Evaluate the following:  
\(4^3\) \hspace{1cm} \(2^5\) \hspace{1cm} \(m^3\) where \(m = -3\)

Answers:  
\(4 \times 4 \times 4 = 64\) \hspace{1cm} \(2 \times 2 \times 2 \times 2 \times 2 = 32\) \hspace{1cm} \((-3)(-3)(-3) = -27\)

The Anatomy (parts) of Exponential Notation

\(2^4\)  
The little number up high is the EXPONENT

\(\downarrow\)  
The bigger portion down low is the BASE. The base can consist of variables and/or numbers.

COMMON MISTAKES

1. Mistaking exponents for multiplication  
   Incorrect: \(2^4 = 2 \times 4 = 8\)  
   Correct: \(2^4 = 2 \times 2 \times 2 \times 2 = 16\)

2. When we say exponential notation out loud: \(2^4\)  
   Incorrect: “Two four”  
   Correct: “Two to the fourth”

3. Exponential notation can only be used in multiplication of terms with the same base, not in addition.  
   Incorrect: \(2 + 2 + 2 + 2 = 2^4\)  
   Correct: \(2 \times 2 \times 2 \times 2 = 2^4\)

4. \(x^2\) vs. \(2x\)  
   Incorrect: \(x \times x = 2x\)  
   Correct: \(x \times x = x^2\) or \(x + x = 2x\)
I. Solving Exponents

EXAMPLE

Solve $7^4$

$7^4 = 7 \times 7 \times 7 \times 7$
$\text{Answer: } 2401$

II. Solving Roots

EXAMPLE

Solve $\sqrt[3]{125}$

$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$

$\forall \sqrt[3]{5 \times 5 \times 5} \rightarrow 5$

Break up the number into its factors by making a factor tree

Now we take out the factors in groups equal to the number of the root we are taking

Once we take out the numbers, the entire group turns into just one number

$\text{Answer } \sqrt[3]{125} = 5$

III. Scientific Notation

LAWS & PROCESSES

Say the following numbers out loud:

602,000,000,000,000,000,000,000

9,460,000,000,000

0.00000000000000000000000299

100,000,000,000

You may be saying the word “zero” a lot because most of us don’t even know the names of these digits, like the first one is “Six hundred two ‘sextillion’”. Who’d have known—even your author had to
look that one up? Believe it or not, each of these four numbers along with hundreds more like them (extremely large or extremely small) is significant, and scientists, educators, and students like you and me are using them every day. See what each means below.

602,000,000,000,000,000,000,000 Avagadro’s #, the number of molecules in a mole (a measure used often in chemistry)

9,460,000,000,000,000 The number of meters light travels in one year also known as a light year. Astronomers and physicists find this useful.

0.0000000000000000000000000299 The mass of a water molecule in kilograms

100,000,000,000 The average number of neurons (also called brain cells) in the human brain.

What if you needed to communicate and use these numbers or others like them that were extremely large or small? Wouldn’t it be helpful to have a shortcut for expressing them? Scientists think so too and call it…

**SCIENTIFIC NOTATION**

<table>
<thead>
<tr>
<th>Standard Notation</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>602,000,000,000,000,000,000,000</td>
<td>6.02 x 10^{23}</td>
</tr>
<tr>
<td>9,460,000,000,000,000</td>
<td>9.46 x 10^{15}</td>
</tr>
<tr>
<td>0.0000000000000000000000000299</td>
<td>2.99 x 10^{-26}</td>
</tr>
<tr>
<td>100,000,000,000</td>
<td>1.0 x 10^{11} or 10^{11}</td>
</tr>
</tbody>
</table>

**Definition of Scientific Notation**
A number is considered in scientific notation if there is only one non-zero digit to the left of the decimal, and it’s being multiplied by a base of 10.
IV. Converting to Scientific Notation

Converting to Scientific Notation
1. Decide which way the decimal will go to get it with only one non-zero digit to the left of the decimal.
2. Count the number of times it moves.
3. Multiply \(X\) 10 to the number. The exponent on the 10 is positive if you moved the decimal to the left, negative if to the right.

**EXAMPLES**

**Convert to Scientific Notation:** 27,500,000,000

We need the number to have one number to the left of the decimal
So we need the decimal needs to go to the left 10 spaces
Because we moved the decimal to the left we will make the number we counted a positive number (left-up)

\[2.75 \times 10^{10}\]

**Answer:** \(2.75 \times 10^{10}\)

**Convert to Scientific Notation:** 0.000000598

We need the number to have one number to the left of the decimal
So we need the decimal to go to the right 7 spaces
Because we moved the decimal to the right we will make the number we counted a negative number

\[5.98 \times 10^{-7}\]

**Answer:** \(5.98 \times 10^{-7}\)
V. Addition/Subtraction of Scientific Notation

Addition/Subtraction of Scientific Notation

1. Set it up
2. Add/Subtract Columns
3. Carry or borrow by 10’s
4. Strongest number wins
5. Put into scientific notation (left – up)

**EXAMPLES**

Find: \((2.6 \times 10^5) + (2.2 \times 10^4)\)

1. First, change them to have the same significant figure
2. Line up by decimal point
3. ADD
4. Change the answer to have one number to the left of the decimal

\[
\begin{align*}
2.60 \times 10^5 &\quad + 2.2 \times 10^4 \\
28.2 \times 10^4 &
\end{align*}
\]

Answer: \(2.82 \times 10^5\)

Subtract: \((8.30 \times 10^8) − (2.4 \times 10^6)\)

1. First, change them to have the same significant figure
2. Line up the decimal points
3. SUBTRACT with the bigger number on top
4. Change the answer to have one number to the left of the decimal

\[
\begin{align*}
83.0 \times 10^6 &\quad - 2.4 \times 10^6 \\
827.4 \times 10^6 &
\end{align*}
\]

Answer: \(8.274 \times 10^8\)

VI. Multiplication with Scientific Notation

Multiplication with Scientific Notation

1. Multiply Decimals
2. Add exponents
3. Put into scientific notation (left – up)
Multiply \((3.42 \times 10^{13}) \times (6.8 \times 10^{3})\)

\[3.42 \times 6.8 = 23.256\]

\[10^{13+3} = 10^{16}\]

\[23,256 \times 10^{16} = 2.3256 \times 10^{17}\]

Answer: \(2.3256 \times 10^{17}\)
### 1.3 EXERCISE SET

#### 1.1 Perform the indicated operation.

1. $0.136 - 0.85$
2. $3.675 + 0.89$
3. $6 \times 27.3$
4. $-0.35 \times 8.6$
5. $0.386 \div 2$
6. $1389 \div -0.3$

#### 1.2 Perform the indicated operation.

7. $\frac{3}{15} + \frac{2}{5}$
8. $\frac{3}{4} - \frac{2}{3}$
9. $-\frac{2}{9} \times \frac{3}{8}$
10. $\frac{5}{3} \times \frac{1}{12}$
11. $-\frac{5}{6} \div \frac{3}{4}$
12. $-\frac{16}{10} \div -\frac{2}{5}$

#### 1.3 Write Exponential Notation and Evaluate.

13. $4 \cdot 4 \cdot 4 \cdot 4$
14. $6 \cdot 6 \cdot 6$
15. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$
16. $(-8)^6$
17. $-2^{10}$
18. $3^5$

#### Find the Roots.

19. $\sqrt[3]{125}$
20. $\sqrt[3]{64}$
21. $\sqrt{36}$
22. $-\sqrt{81}$
23. $\sqrt[5]{7776}$
24. $\sqrt[4]{625}$

#### Convert to Scientific Notation.

25. $15,625,000$
26. $-0.0000657$
27. $481,516$

#### Evaluate.

28. $2.789 \times 10^8$
29. $8.97 \times 10^{-10}$
30. $4.81516 \times 10^2$

#### Add or Subtract.

31. $1.658 \times 10^5 + 2.56 \times 10^6$
32. $4.658 \times 10^{-8} + 3.897 \times 10^{-7}$
33. $48.9 \times 10^6 - 3.25 \times 10^9$
34. $-10.78 \times 10^4 + 8.63 \times 10^5$

#### Preparation.

35. Rewrite as a decimal: a) $\frac{59}{100}$ b) $\frac{73}{100}$ c) $\frac{0.3}{100}$
Answers

1. $-0.714$
2. $4.565$
3. $163.8$
4. $-3.01$
5. $0.193$
6. $-4630$
7. $\frac{3}{5}$
8. $\frac{1}{12}$
9. $-\frac{1}{12}$
10. $\frac{5}{36}$
11. $-\frac{10}{9}$
12. $4$
13. $4^4, 256$
14. $6^3, 216$
15. $390625$
16. $262144$
17. $-1024$
18. $243$
19. $5$
20. $4$
21. $6$
22. $-9$
23. $6$
24. $5$
25. $1.5625 \times 10^7$
26. $-6.57 \times 10^{-5}$
27. $4.81516 \times 10^5$
28. $278,900,000$
29. $0.00000000897$
30. $481.516$
31. $2.7258 \times 10^6$
32. $4.3628 \times 10^{-7}$
33. $-3.2011 \times 10^9$
34. $7.552 \times 10^5$
35. In class.
### 1.4 Percents

#### OBJECTIVES
- Convert percents into decimals and fractions and vice versa
- Solve percent problems with “of”

#### REVIEW OF ARITHMETIC AND VARIABLES (Overview)

Percents are such a huge part of our society, what with sales tax, income tax, discounts, sales, etc. They are everywhere, and they aren’t too hard to understand. They are really just another way of writing fractions or decimals!

#### LAWS & PROCESSES

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#### EXAMPLES

<table>
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<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
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</thead>
<tbody>
<tr>
<td>(\frac{1}{3})</td>
<td>0.33</td>
<td>33%</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>(\frac{7}{5})</td>
<td>1.40</td>
<td>140%</td>
</tr>
</tbody>
</table>
PERCENT: Percent can be broken up into two words: “per” and “cent” meaning per hundred, or in other words, hundredths.

Example:
\[
\frac{7}{100} = .07 = 7\%
\]
\[
\frac{31}{100} = .31 = 31\%
\]
\[
\frac{53}{100} = .53 = 53\%
\]

Notice the shortcut from decimal to percents: move the decimal to the right two places.

I. Converting Percents

**Converting Percents**

1. If it is a fraction, solve for decimals
2. Move decimal 2 places
3. “OF” means times

**EXAMPLES**

1. Convert .25 to percents
   \[
   .25 = 25\%
   \]
   Move the decimal two places to the right because we are turning this into a percent
   Answer: 25%

2. What is \(\frac{5}{32}\) as a percent?
   \[
   5 \div 32 = .15625
   \]
   Turn the fraction into a decimal by dividing
   \[
   .15625 = 15.625\%
   \]
   Move the decimal two places to the right because we are turning this into a percent
   Answer: 15.625%

3. Convert 124% to decimals
   \[
   124\% = 1.24
   \]
   Move the decimal two places to the left because we are turning this into a decimal
   Answer: 1.24
II. Solving Percent Problems with “of”

**Solving Percent Problems with “of”**

1. Turn percent into a decimal
2. Multiply the two numbers together

**COMMON MISTAKES**

The most important thing that you should know about percents is that they **never stand alone**. If I were to call out that I owned 35%, the immediate response is, “35% of what?”

Percents always are a percent of something. For example, sales tax is about 6% or 7% of your purchase. Since this is so common, we need to know how to figure this.

If you buy $25 worth of food and the sales tax is 7%, then the actual tax is 7% of $25.

\[ .07 \times 25 = $1.75 \]

**EXAMPLES**

What is 25% of 64?

- 25\% = .25 \hspace{1cm} \text{Turn the percent into a decimal}
- \[ .25 \times 64 = 16 \hspace{1cm} \text{Multiply the two numbers together} \]
- Answer: 16

What is 13% of $25?

- 13\% = .13 \hspace{1cm} \text{Turn the percent into a decimal}
- \[ .13 \times 25 = 3.25 \hspace{1cm} \text{Multiply the two numbers together} \]
- Answer: $3.25

What is 30% of 90 feet?

- 30\% = .30 \hspace{1cm} \text{Turn the percent into a decimal}
- \[ .30 \times 90 = 27 \hspace{1cm} \text{Multiply the two numbers together} \]
- Answer: 27 feet
Perform the indicated operation.

1. $25.85 - 12.9$
2. $36.02 + .75$
3. $.25 \times 14$
4. $-2.56 \times .32$
5. $36.15 \div .3$
6. $-81 \div .09$

Perform the indicated operation.

7. $\frac{5}{6} + \frac{3}{8}$
8. $\frac{8}{12} - \frac{1}{8}$
9. $-\frac{1}{4} \times \frac{5}{8}$
10. $\frac{2}{3} \times \frac{6}{12}$
11. $-\frac{5}{8} \div \frac{55}{56}$
12. $-\frac{21}{25} \div -\frac{57}{60}$

Solve.

13. $\sqrt[1]{81}$
14. $\sqrt[3]{512}$
15. $\sqrt[5]{-59049}$
16. $8.63 \times 10^8 \cdot 2.87 \times 10^{-6}$

Convert to either decimal notation or scientific notation.

17. $2,530,000,000$
18. $3.86 \times 10^{-7}$

Convert to decimal notation.

19. $73\%$
20. $13.25\%$
21. $94\%$
22. $30\%$
23. $255\%$
24. $.067\%$

Convert to decimal notation (round to five decimal places) and then to a percent.

25. $\frac{3}{8}$
26. $\frac{18}{120}$
27. $\frac{23}{56}$
28. $\frac{33}{96}$
29. $\frac{66}{130}$
30. $\frac{49}{52}$
31. $\frac{9}{14}$
32. $\frac{5}{36}$

Evaluate.

33. $15\%$ of 26
34. $80\%$ of 294
35. $47\%$ of 47
36. $63\%$ of 2,436
37. $9\%$ of 50
38. $3\%$ of 200

Preparation.

39. Read some of Section 1.5 and then round to the nearest hundred:
   a) 371    b) 1,799,440    c) 98.4341
Answers

1. 12.95
2. 36.77
3. 3.5
4. \(-0.8192\)
5. 120.5
6. \(-900\)
7. \(29/24\)
8. \(13/24\)
9. \(-5/32\)
10. \(1/3\)
11. \(-7/11\)
12. \(84/95\)
13. 3
14. 8
15. \(-9\)
16. \(2.47681 \times 10^3\)
17. \(2.53 \times 10^9\)
18. \(0.000000386\)
19. \(0.73\)
20. \(0.1325\)
21. \(0.94\)
22. \(0.3\)
23. \(2.55\)
24. \(0.00067\)
25. \(0.375, 37.5\%\)
26. \(0.15, 15\%\)
27. \(0.41071, 41.071\%\)
28. \(0.34375, 34.375\%\)
29. \(0.50769, 50.769\%\)
30. \(0.94231, 94.231\%\)
31. \(0.64286, 64.286\%\)
32. \(0.13889, 13.889\%\)
33. 3.9
34. 235.2
35. 22.09
36. 1,534.68
37. 4.5
38. 6
39. In class.
Now that you have finished arithmetic, and you have seen how some problems may get long and tedious, you may understand why some folks choose to estimate and round numbers.
1. About how many dollars will you have if you start with $15,474, take 48% of it, and then subtract off $155 to pay a bill, and then receive $49?

\[
\begin{align*}
15,472 & \approx 15,500 & \text{Rounding} \\
48\% & \approx 0.48 \approx 0.50 & \text{Estimation} \\
\text{so} & \quad 15,500 \\
\times & \quad 0.50 \\
\$7750.00 & \approx 0.50 \\
\$7750 & \approx \$155 + \$49 = \text{Order of Operations} \\
\$7750 & \approx \$155 + \$49 \\
\approx \$7600 & \approx \$7600 \\
\end{align*}
\]

Answer: \$7600

B. Rounding and Estimation; Order of Operations

**Definitions & Basics**

**Rounding:** In rounding, we decide to not keep the exact number that someone gave us. For example:

If I have $528.37 in the bank, I might easily say that I have about $500. I have just rounded to the nearest hundred.

On the other hand, I might be a little more specific and say that I have about (still not exact) $530. I have just rounded to the nearest ten.

**Estimation:** Once rounding is understood, it can be used as a great tool to make sure that we have not missed something major in our computations. If we have a problem like:

\[
3,427,000 \\
\times 87.3
\]

We could see about where the answer is if we estimate first:

- Round each number to the greatest value you can
- $3,000,000$ \\
- $\times 90$

and our answer will be around 270,000,000

We should note that the real answer is: 299,177,100 but the estimation will let us know that we are in the right ball park. It ensures that our answer makes sense.
INEQUALITIES: You know how an equal sign works – it is placed between two things that mean the same thing. If you wanted to show that two things were not equal, you would use an inequality:

<table>
<thead>
<tr>
<th>Symbols of Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>&lt;</code> Less than</td>
</tr>
<tr>
<td><code>&gt;</code> Greater than</td>
</tr>
<tr>
<td><code>≤</code> Less than or equal to</td>
</tr>
<tr>
<td><code>≥</code> Great than or equal to</td>
</tr>
</tbody>
</table>

The symbol always points to the smaller number. You can also remember that the two lines are farther apart on the BIGGER side, and close together on the smaller side, so the distances between the two lines is related to how big the number is.

Each symbol can be flipped around as long as the numbers on either side are also flipped around. For example, 7 > -4 and -4 < 7 mean the same thing, and -4 is always on the small side of the sign.

ABSOLUTE VALUE: Another symbol to be aware of is the absolute value symbol. To find the “absolute value” means to find out how far away from zero a number is, like on a number line. For example, the number 5 is 5 units away from zero. The number -5 is also five units away from zero (just in the opposite direction).

**Examples:** | 13 | = 13 | -9 | = 9 | -43 | = 43

P.E.M.D.A.S.: An acronym to help remember the order of doing things.

- **P** = parenthesis
- **E** = exponents
- **M.D.** = multiplication and division
- **A.S.** = addition and subtraction

These are grouped like this because they are so similar they should be done at the same time.

I. **Rounding**

**EXAMPLES**

2. **Round $4,278.23 to the nearest hundred**

   - $4,300.00
   - $4,200.00
   - $4,278.23 ≈ $4,300.00

   Decide if our number is closer to the nearest hundred above the number or below the number
   - Change our number to the one it is closer to

   **Answer:** $4,300.00
II. Estimation

**Estimation**
1. Round to the highest value
2. Do the easy problem

**EXAMPLES**

3. Multiply by rounding: \(986.7 \times 4.9\)

\[
\begin{align*}
986.7 & \approx 1,000 & \text{Round the numbers} \\
4.9 & \approx 5 \\
1,000 \times 5 & = 5,000 & \text{Multiply the rounded numbers together} \\
986.7 \times 4.9 & \approx 5,000 & \text{Our answer for } 986.7 \times 4.9 \text{ will be around } 5,000 \\
\end{align*}
\]

Answer: 5,000

III. Inequalities

**EXAMPLES**

4. Write an equivalent expression: \(38 > 14\)

\[
\begin{align*}
38 > 14 & \quad \text{Flip the sign and the numbers} \\
14 & < 38 \\
\end{align*}
\]

Answer: \(14 < 38\)

IV. Absolute Value

**EXAMPLES**

5. Find the absolute value of the following: \(8, -16, -\frac{2}{3}\)

\[
\begin{align*}
|8| & = 8 \\
|-16| & = 16 \\
|\frac{-2}{3}| & = \frac{2}{3} \\
\end{align*}
\]

Answer: 8, 16, \(\frac{2}{3}\)
V. Getting the Right Order

Remember your order of operations, P.E.M.D.A.S. Always follow this order when simplifying or solving problems. The last two steps, Multiplication/Division, and Addition/Subtraction, should be done in order from left to right so that nothing (especially negatives) is missed.

**EXAMPLES**

6  **Simplify: $2 + 3 \times 4 - 5$**

If you were to read that from left to right you would first add the 2 and the 3 to get 5 and then multiply by 4 to get 20.

But remember that multiplication is a shorthand way of writing repeated addition. Technically we have:

\[
2 + 3 \times 4 - 5 = 2 + 4 + 4 + 4 - 5 = 9.
\]

This is the right answer. We need to take care of the multiplication as a group, before we can involve it in other computations.

**Multiplication is done before addition and subtraction.**

**Answer: 9**

7  **Simplify: $4 \times 3^2 - 7 \times 2 + 4$**

**Take care of exponents**

\[
4 \times 3^2 - 7 \times 2 + 4 = 4 \times 9 - 7 \times 2 + 4 = 36 - 14 + 4 = 22 + 4 = 26.
\]

**Answer: 26**

**COMMON MISTAKES**

That last question presents an opportunity to make a common mistake while trying to simplify. Many suggest that addition happens before subtraction (PEMDAS, right?)

\[
36 - 14 + 4 = 36 - 18 = 18
\]

But really, subtraction and addition are the same thing, so you can’t do one before the other. Read it left to right.

\[
36 - 14 + 4 = 22 + 4 = 26
\]

The same thing goes for multiplication and division, since division is the same thing as multiplication.

Section 1.5
Perform the indicated operation.

1. \(-5.635 - 9.83\)  
2. \(-15.892 + .873\)  
3. \(.18 \times -.63\)

4. \(-23.65 \times -.05\)  
5. \(-25.15 \div .005\)  
6. \(8.6 \div .07\)

Perform the indicated operation.

7. \(\frac{6}{25} + \frac{5}{8}\)  
8. \(\frac{8}{120} - \frac{3}{5}\)  
9. \(-\frac{4}{25} \times \frac{5}{8}\)

10. \(\frac{1}{3} \times \frac{2}{12}\)  
11. \(\frac{6}{11} \div \frac{42}{55}\)  
12. \(\frac{23}{25} \div \frac{46}{60}\)

Solve, convert or simplify.

13. \(\sqrt{36}\)  
14. \(\sqrt[3]{343}\)

15. \(6.89 \times 10^9\)  
16. \(3.5 \times 10^7 + 6.8 \times 10^5\)

17. \(3.86 \times 10^{-7} + 5.6 \times 10^{-6}\)  
18. \(12.6 \times 10^5 \cdot 1.056 \times 10^6\)

Convert to decimal notation (round to four decimal places) and then to a percent.

19. \(\frac{5}{16}\)  
20. \(\frac{24}{5}\)  
21. \(\frac{83}{20}\)  
22. \(\frac{33}{80}\)  
23. \(\frac{75}{86}\)  
24. \(\frac{88}{96}\)

Find the following.

25. 24% of 92  
26. 17% of 85  
27. .3% of 365

Round to the nearest tenth.

28. 42.142956  
29. .47937  
30. 13,693.639  
31. 284.359432

Round to the nearest hundred.

32. 23,978.74  
33. 74.90  
34. 149.99  
35. 1,999,999.99

Write an equivalent expression.

36. \(6 \geq 1.5\)  
37. \(2,349 < 4,991\)  
38. \(-16 > -24\)

Find the absolute value.

39. \(|-17|\)  
40. \(|8-14|\)  
41. \(|3(4 - 2)|\)
Follow order of operations to solve.

42. \[216 \cdot 6^3 \div 6^2\]  
43. \[\frac{2}{3} + 18 \div 3\]  
44. \[5^2 - (5 + 6) \cdot 7\]  
45. \[11 - 26 + 27 \div 3\]  
46. \[\frac{6}{8} \cdot \frac{2}{3} + 2\]  
47. \[2^3 - 5 \cdot 3 + 8 \cdot 10 \div 2\]  
48. \[4 \div 8 + 15 + (16 - 23) \times 42\]

Preparation.

49. Estimate the following product: \[2(\pi)(7.82)\] where \(\pi\) is a number equal to 3.14156265…..
Answers

1. $-15.465$
2. $-15.019$
3. $-0.1134$
4. $1.1825$
5. $-5030$
6. $122.857$
7. $\frac{173}{200}$
8. $-\frac{8}{15}$
9. $-\frac{1}{10}$
10. $\frac{1}{18}$
11. $\frac{5}{7}$
12. $-\frac{6}{5}$
13. $6$
14. $7$
15. $6,890,000,000$
16. $3.568 \times 10^7$
17. $5.986 \times 10^{-6}$
18. $1.33056 \times 10^{12}$
19. $0.3125, 31.25\%$
20. $48,480\%$
21. $4.15, 415\%$
22. $0.4125, 41.25\%$
23. $0.8721, 87.21\%$
24. $0.9167, 91.67\%$
25. $22.08$
26. $14.45$
27. $1.095$
28. $42.1$
29. $0.5$
30. $13,693.6$
31. $284.4$
32. $24,000$
33. $100$
34. $100$
35. $2,000,000$
36. $1.5 \leq 6$
37. $4,991 > 2,349$
38. $-24 < -16$
39. $17$
40. $6$
41. $6$
42. $1296$
43. $\frac{20}{3}$
44. $-52$
45. $-6$
46. $\frac{5}{2}$ or $2\frac{1}{2}$
47. $33$
48. $-278.5$
49. In class.
In mathematics, there are a lot of times when we don’t know something and we have to represent what we don’t know with a symbol. These are often letters, which we refer to as **variables**.

Formulas are used in many occupations. Here are a few examples.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Practical Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = vt + x_o )</td>
<td>Physics – finding position</td>
</tr>
<tr>
<td>( P = 4v^2 )</td>
<td>Medicine – pressure in the heart</td>
</tr>
<tr>
<td>( A = P \left(1 + \frac{r}{n}\right)^{nt} )</td>
<td>Finances – bank account balance with compound interest</td>
</tr>
</tbody>
</table>
VARIABLES: These symbols or letters, actually represent numbers, but the numbers can change from time to time, or vary. Thus are they called variables.

Example: Tell me how far you would be walking if you were to walk around this rectangle.

```
24 ft
15 ft           15 ft
```

24 ft

It appears that to get all the way around it, we simply add up the numbers on each side until we get all the way around.

\[24 + 15 + 24 + 15 = 78\]

So if you walked around a 24ft × 15ft rectangle, you would have completed a walk of 78 ft. I bet we could come up with the pattern for how we would do this all of the time. Well, first of all, we just pick general terms for the sides of the rectangle:

```
length
width            width
width
length
```

Then we get something like this:

Distance around the rectangle = length + width + length + width

Let's try and use some abbreviations. First, “perimeter” means “around measure”. Substitute it in:

Perimeter = length + width + length + width

Let's go a bit more with just using the first letters of the words:

\[P = l + w + l + w\]

Notice now how each letter stands for a number that we could use. The number can change from time to time. This pattern that we have created to describe all cases is called a formula.

FORMULA: These are patterns in the form of equations and variables, often with numbers, which solve for something we want to know, like the perimeter equation before, or like:

Area of a rectangle: \[A = B \times H\]
**Volume of a Sphere:** \( V = \frac{4}{3} \pi r^3 \)

**Pythagorean Theorem:** \( a^2 + b^2 = c^2 \)

**COMMON GEOMETRIC FORMULAS:** Now that you understand the idea, these are some basic geometric formulas that you need to know:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>( P = 2l + 2w ) ( A = lw )</td>
<td>( P ) is the perimeter ( l ) is the length ( w ) is the width ( A ) is the Area</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>( P = 2a + 2b ) ( A = bh )</td>
<td>( P ) is the perimeter ( a ) is a side length ( b ) is the other side length ( h ) is height ( A ) is the Area</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>( P = b+a+B+d ) ( A = \frac{1}{2}h(B+b) )</td>
<td>( P ) is perimeter ( b ) is the little base ( B ) is the big base ( a ) is a leg ( h ) is height ( d ) is a leg ( A ) is the Area</td>
</tr>
<tr>
<td>Triangle</td>
<td>( P = s_1+s_2+s_3 ) ( A = \frac{1}{2}bh )</td>
<td>( P ) is the perimeter ( h ) is height ( b ) is base ( A ) is the Area</td>
</tr>
<tr>
<td>Triangle</td>
<td>( a + b + c = 180 )</td>
<td>( a ) is one angle ( b ) is another angle ( c ) is another angle</td>
</tr>
<tr>
<td>Rectangular Solid</td>
<td>( SA = 2lh+2wh+2h ) ( V = lwh )</td>
<td>( l ) is the length ( h ) is the height ( w ) is the width ( SA ) is the Surface Area ( V ) is volume</td>
</tr>
</tbody>
</table>
| **Circle** | $C = 2\pi r$  
$A = \pi r^2$  
$r$ is the radius of the circle  
$A$ is the area inside the circle. |
|---|---|
| **Cylinder** | $LSA = 2\pi rh$  
$SA = 2\pi rh + 2\pi r^2$  
$V = \pi r^2 h$  
$LSA$ is Lateral Surface Area = Area just on the sides  
$h$ is the height  
$SA$ is total surface area  
$\pi$ is a number, about 3.14159 . . .  
it has a button on your calculator  
r is the radius of the circle  
$V$ is Volume |
| **Cone** | $LSA = \pi rl$  
$SA = \pi r^2 + \pi rl$  
$V = \frac{1}{3} \pi r^2 h$  
$h$ is the height  
r is the radius of the circle  
l is the slant height  
$\pi$ is a number, about 3.14159 . . .  
it has a button on your calculator  
$SA$ is total surface area  
$LSA$ is Lateral Surface Area = Area just on the sides  
$V$ is Volume |
| **Sphere** | $SA = 4\pi r^2$  
$V = \frac{4}{3} \pi r^3$  
r is the radius  
$SA$ is the surface area  
$V$ is the Volume |
I. Shapes and Substitutions

The list of geometric formulas will be helpful in the following types of problems. You will be given information about a shape and asked for a perimeter or a volume, etc. For these problems, use the appropriate formula and fill in the information that you know. Then do the math to find what you don't know.

**EXAMPLES**

2. What is the volume of a cylinder when the height is 6 cm and the radius is 2 cm?

Remember that \( V = \pi r^2 h \)

First draw a picture and label the appropriate sides

Write out the equation while remembering to substitute for \( r \) and substitute for \( h \)

Solve using order of operations

**Answer:** \( V = 75.4 \text{cm} \)

You will also be asked to do problems where two variables are not known, but you do know a relationship between the two variables. In this situation, a substitution is necessary. See the following example that uses substitution.

3. What is the area of your lawn if the length is 25 feet longer than the width and the width is 35 feet?

Draw a picture of the situation and label the appropriate sides

Write out an equation for the area of the lawn while remembering to substitute for "w" and substitute for "l"

Solve using order of operations

**Answer:** \( A = 2100 \text{ft}^2 \)
The depth of a rectangular pool is 17 feet less than half of the length and the width is 20 feet less than the length. If the pool is 54 feet long, how much water would you need to fill up a rectangular pool? Remember that $V = lwh$

Draw a picture of the situation and label the appropriate sides.

Write out an equation for the volume of the pool in terms of what you know, this case being in terms of “$l$”.

Substitute for “$l$”

Solve using order of operations

Answer: $18360 \, ft^3$

II. Distance, Rate, and Time

If you were traveling at 40mph for 2 hours, how far would you have traveled? Well, most of you would be able to say 80 mi. How did you come up with that? Multiplication:

$$ (40) \cdot (2) = 80 $$

(rate of speed) \cdot (time) = distance

or in other words: $rt = d$ where $r$ is the rate, $t$ is the time, and $d$ is the distance.

Distance, Rate, and Time

$$ rt = d $$

$r = \text{rate}, \ t = \text{time}, \ d = \text{distance}$

EXAMPLES

Stacey traveled 3 hours while going 27 mph. using the formula $rt = d$ determine the distance that she traveled.

Write down what we information we have been given

Decide what we are trying to figure out

Plug all of the information that we already know into the equation we are given

Solve for the piece of information that we don’t know yet

Answer: 81 miles
III. Calculating Taxes and Discounts

**TAX:** If you bought something for $5.50 and there was a 8% sales tax, you would need to find 8% of $5.50 to find out how much tax you were being charged.

\[
.44 = .08(5.50)
\]

Amount of Tax = (Tax rate) \cdot (Purchase amount)

or in other words: \[ T = rP \]

where T is tax

r is rate of tax

P is the original price

**DISCOUNT:** If you were going to buy something for $60.99, and there was a 15% discount, you would like to know 15% of $60.99 to find out how much you were going to save.

\[
9.15 = .15(60.99)
\]

Amount of Discount = (Discount rate) \cdot (Original Price)

or in other words: \[ D = rP \]

where D is discount

r is rate of discount

P is the original price.

**EXAMPLES**

6. If you want to buy a $759 computer with 8% sales tax, how much tax will you end up paying?

\[
T = rP
\]

\[
T = \frac{8}{100} \times 759
\]

\[
T = 60.72
\]

Answer: You would pay $60.72 in sales tax

7. How much will Alice save on a pair of shoes that are worth $92 but are on sale for 20% off?

\[
D = rP
\]

\[
D = \frac{20}{100} \times 92
\]

\[
D = 18.40
\]

Answer: Alice will save $18.40
IV. Simple Interest

This formula is very similar to another one that involves simple interest. If you invested a principal amount of $500 at 9% interest for three years, the amount of interest would be given by the formula:

\[ I = Prt \]

where

- \( I \) is the interest earned
- \( P \) is the principal amount (starting amount)
- \( r \) is the interest rate
- \( t \) is the time that it is invested.

**EXAMPLES**

Mindy sets up a savings plan that gives her simple interest of 7% per year. If she invests $750, how much interest will she earn in 10 years?

\[ I = Prt \]
\[ r = 7\% \]
\[ P = 750 \]
\[ t = 10 \text{ years} \]
\[ I = ? \]

\[ I = (750)(.07)(10) \]
\[ I = 525 \]

Answer: $525

V. Temperature Conversions of Celsius and Fahrenheit

Most of us know that there is a difference between Celsius and Fahrenheit degrees, but not everyone knows how to get from one to the other. The relationship is given by:

\[ C = \frac{5}{9} (F - 32) \]

where

- \( F \) is the degrees in Fahrenheit
- \( C \) is the degree in Celsius

A simple manipulation of the same formula can give you the process for switching back the other way:

\[ F = \frac{9}{5} C + 32 \]
9. If your thermometer in your car says it is 94º Fahrenheit, what is the temperature in Celsius?

Write down what we information we have been given

\[ C = \frac{5}{9}(F - 32) \]

Decide what we are trying to figure out

\[ F = 94º \]

Plug all of the information that we already know into the equation we are given

\[ C = \frac{5}{9}(94 - 32) \]

Solve for the piece of information that we don’t know yet

\[ C = \frac{5}{9}62 \]

\[ C = 34.4 \]

Answer: 34.4º Celsius

10. If it is 4º Celsius outside, what is the temperature in Fahrenheit?

Write down what we information we have been given

\[ F = \frac{9}{5}C + 32 \]

Decide what we are trying to figure out

\[ C = 4º \]

Plug all of the information that we already know into the equation we are given

\[ F = \frac{9}{5}(4) + 32 \]

Solve for the piece of information that we don’t know yet

\[ F = 7.2 + 32 \]

\[ F = 39.2 \]

Answer: 39.2º Fahrenheit

Section 1.6
Perform the indicated operation.

1. $7.2 + 13.258$

2. $18.6794 - 237.58$

3. $0.298 \times 1.4$

Perform the indicated operation.

4. $\frac{4}{5} \times \frac{15}{16}$

5. $\frac{15}{21} \div \frac{5}{42}$

6. $\frac{7}{120} \div -\frac{21}{40}$

Solve, convert or simplify.

7. $\sqrt[4]{81}$

8. $4.81516 \times 10^{-7}$

9. $9.52 \times 10^9 + 2.59 \times 10^7$

Convert to decimal notation (round to four decimal places) and then to a percent.

10. $\frac{6}{19}$

11. $\frac{15}{4}$

12. $\frac{126}{3150}$

Round to the nearest hundredth.

13. $163.69387$

14. $0.01982465$

Round to the nearest thousand.

15. $235,724.98$

16. $9,598,482,099.99$

Follow order of operations to solve.

17. $3^4 + (5 \times (8 \div 4) - 3)$

18. $4 \times (8 + 15 \div (16 - 23) \times 42)$

Evaluate the expression with the given variables.

19. $4x + t$: when $x=4$ and $t=16$

20. $19x - 47y$: when $x=-4$ and $y=3$

21. $x + 2y - 3z$: when $x=18$; $y=3$; $z=20$

22. $\frac{4}{5}x + \frac{2}{7}y$: when $x=5$ and $y=7$

Find the missing variable. (Note: If you don’t use the $\pi$ button on your calculator, your answer will differ slightly)

23. For a triangle

24. For a cone

25. For a sphere

$b = 4$ in

$r = 3.8$ m

$V = ?$

$h = 7$ in

$l = 5.1$ m

$SA = ?$

$A = ?$

26. I have a rectangular sand box whose length is 4 more than its width. If the width is 12 ft, what is the perimeter of the sand box?

27. What is the volume of a cylinder whose height is 3 cm less than twice its radius? The radius is 4 cm.
Find the missing variable.

28. Distance:
   \[ r = 75 \text{ mph} \]
   \[ t = 5 \text{ hrs} \]
   \[ d = ? \]

29. Tax:
   \[ r = 6\% \]
   \[ P = $29.95 \]
   \[ T = ? \]

30. Discount:
   \[ r = 30\% \]
   \[ P = $48 \]
   \[ D = ? \]

31. Interest:
   \[ P = $2500 \]
   \[ r = 3.5\% \]
   \[ t = 2 \text{ years} \]
   \[ I = ? \]

32. Temperature:
   \[ F = 88^\circ \]
   \[ C = ? \]

33. Temperature:
   \[ C = -12^\circ \]
   \[ F = ? \]

Preparation.

34. Read some of 1.7 and then simplify the following:
   a) \[ 2x + 4x \]
   b) \[ 8 - 4 + 3y - 2y \]
Answers
1. 20.458
2. −218.9006
3. 0.4172
4. $\frac{3}{4}$
5. 6
6. $-\frac{1}{9}$
7. 3
8. 0.00000481516
9. 9.5459 \times 10^9
10. .3158, 31.58%
11. 3.75, 375%
12. .04, 4%
13. 163.69
14. .02
15. 236,000
16. 9,598,482,000
17. 88
18. −328
19. 32
20. −217
21. −36
22. 6
23. 14 \text{ in}^2
24. 106.25 \text{ m}^2
25. 17157.28 \text{ cm}^3
26. 56 \text{ ft}
27. 251.33 \text{ cm}^3
28. 375 \text{ miles}
29. $1.80$
30. $14.40$
31. $175$
32. 31.11° C
33. 10.4 ° F
34. In class.
Whenever we deal with variables, and especially a lot of them, we need ways of simplifying and using them. There are rules that define what we can do, what is allowed, and give us ideas of how to solve for what we don’t know. These are the ones you need to know now.

### Examples

<table>
<thead>
<tr>
<th>Example</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3xy + 7t = 7t + 3xy$</td>
<td>Commutative Addition</td>
</tr>
<tr>
<td>$3(x5) = (3x)5$</td>
<td>Associative Multiplication</td>
</tr>
<tr>
<td>$7(x - 4) = 7x - 28$</td>
<td>Distributive</td>
</tr>
</tbody>
</table>
The Rules: These are the different properties that you use when simplifying an equation. They are simple rules that you have been using with addition and multiplication; all we have done is given them names and applied them to variables.

<table>
<thead>
<tr>
<th>Name</th>
<th>Operation</th>
<th>What it does</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Commutative**           | Addition  | We can swap numbers around addition                     | 5 + 7
                                          |                        | 7 + 5 are the same.    |
                                          |            |                                                        | 2x + 3y
                                          |                        | 3y + 2x are the same.  |
| **Commutative**           | Multiplication | We can swap numbers around multiplication       | 7xtzy
                                          |                        | x7tzy are the same.    |
| **Associative**           | Addition  | We can begin addition with any of the entries.         | 7+(5+1)
                                          |                        | (7+5)+1 are the same.  |
                                          |            |                                                        | (x+5)+9
                                          |                        | x+(14) are the same.   |
| **Associative**           | Multiplication | We can begin multiplication with any of the entries | 7⋅(3⋅2)
                                          |                        | are the same           |
                                          |            |                                                        | (7⋅3)⋅2               |
| **Associative and**       | Either    | We can multiply (or add) in any order we want to.      | 6+0=6
                                          | Commutative            |                        | x+0=x                  |
                                          | or Addition|                                                        |                       |
| **Identity**              | Addition  | 0 is invisible in addition                             | 6⋅1 = 6
                                          |                        | x⋅1 = x                |
| **Identity**              | Multiplication | 1 is invisible in multiplication                      | 6 -6
                                          |                        | 3t -3t are opposites  |
                                          |            |                                                        | -17 17 add to zero.   |
| **Inverse**               | Addition  | Opposite when adding “gets rid” of the number           | -5 -\frac{1}{2}      |
                                          |            |                                                        | -\frac{2}{3} -\frac{1}{2} multiply to 1. |
                                          |            |                                                        | 17 1/17                |
| **Inverse**               | Multiplication | Opposite when multiplying (inverses) “gets rid” of the number | -\frac{5}{7} -\frac{1}{7} |
                                          |            |                                                        | -\frac{3}{5} -\frac{1}{5} |
                                          |            |                                                        | 6 \cdot \frac{2}{17} multiply to 1. |
| **Distributive**          | Both      | Jump numbers into parentheses                          | 6(43) = 6(40+3) = 6(40) + 6(3) |
                                          |            |                                                        | 7(2x - 5) = 14x - 35  |
I. Identifying Properties

1. Identify the following properties

   \[(79.2 \times 23) \times 5 = 79.2 \times (23 \times 5)\]  
   Answer: Associative Property of Multiplication

   \[12x + 0 = 12x\]  
   Answer: Identity Property of Multiplication

   \[3(2t - 6) = 3(2t) - 3(6)\]  
   Answer: Distributive Property

   \[13.7 + 3 + 19 = 19 + 13.7 + 3\]  
   Answer: Commutative Property of Addition

II. Using the Properties

2. Simplify: \(6 \times \frac{1}{6} + 4 - 4 + 2(7 + 3)\)

   \(\left(6 \times \frac{1}{6}\right) + 4 - 4 + 2(7 + 3)\)  
   First use the inverse property of multiplication

   \(1 + 4 - 4 + 2(7 + 3)\)  
   Next use the inverse property of addition

   \(1 + 0 + 2(7 + 3)\)  
   Then we can use the identity property of addition to make the 0 invisible

   \(1 + 2(7 + 3)\)  
   Now use the distributive property

   \(1 + 2(14) + 2(3)\)  
   Solve

   \(1 + 28 + 6 = 35\)  
   Answer: 35

III. Simplifying with Variables

Addition and subtraction are very straightforward with just numbers, but what about with variables? Remember the example from Section 1:

Example: If someone asks you

5 sheep + 2 sheep = ?

you would be able to tell them 7 sheep.
What if they asked you 5 sheep + 2 penguins = ?
We really can’t add them together, because they aren’t like things.

It works the same way with variables. Just think of “sheep” and “penguins” in this example as variables. In fact, let’s say s stands for sheep, and p stands for penguins. We can re-write these two equations using variables:

5 sheep + 2 sheep = 7 sheep
5s + 2s = 7s

5 sheep + 2 penguins = 5s + 2p

So adding and subtracting like terms works the same way with variables as it does with sheep, penguins, fractions with common denominators, and other quantities with like terms.

EXAMPLES

3  Simplify 3x + 9x – 7y

3x + 9x – 7y
12x – 7y  Combine like terms
Answer: 12x – 7y

4  Simplify 3(2a – 4b) + 5(2b)

3(2a – 4b) + 5(2b)
6a – 12b + 10b  Distributive property and multiplication
6a – 2b  Combine like terms
Answer: 6a – 2b


2[2 – [3(4) + (–4)] + [2(7) – 3(–2)]]  Start by solving inside the parenthesis first
2[2 – [12 – 4] + [14 – (–6)]]  Next solve inside the brackets
2[2 – [8] + [20]]
2{14} = 28  Now solve inside the curly bracket
Distribute into the curly bracket
Answer: 28
IV. Factoring

FACTORS: These values (numbers or variables) when multiplied equal the given value OR things that can go into the given value and yield a whole # quotient. Basically a factor is a number that can multiply to the number you are given.

Example: If asked the factors of 24 (the given number), they are 1&24, 2&12, 3&8, 4&6 or 1,2,3,4,6,8,12,24

COMMON FACTOR: A factor that two or more terms have in common.

Example: Two terms: 24 and 36.
Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36
The numbers in boxes are the common factors between 24 and 36.

GREATEST COMMON FACTOR: This is the biggest factor that all terms share in common.

Example: The Greatest Common Factor of 24 and 36 is 12, because it is a factor that they share and it is also the greatest.

Factor as a verb can mean several different things, which you will learn more about in Chapter 5. For now, to factor an expression just means to pull out the greatest common factor of each term in the expression. This means you will divide each term by the greatest common factor and then re-write the expression with that common factor on the outside of a set of parentheses and the “left-overs” on the inside of the parenthesis. It’s a little bit like the distributive property in reverse. Example 6 will explain more.

EXAMPLES

6. Find the greatest common factor of 16 and 48.

Factors of 16: 1,2,4,8,16
Factors of 48: 1,2,3,4,6,8,12,16,24,48

Factors of 16: 1,2,4,8,16
Factors of 48: 1,2,3,4,6,8,12,16,24,48

Answer: 16
**Factor: 3x + 6**

<table>
<thead>
<tr>
<th>Factors of 3: 1, 3</th>
<th>Find the greatest common factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors of 6: 1, 2, 3, 6</td>
<td>Divide each term by the greatest common factor</td>
</tr>
</tbody>
</table>

\[
\frac{3x + 6}{3} = \frac{3(x + 2)}{3}
\]

Write the expression with the common factor on the outside of the parentheses and the “left-overs” inside

**Answer:** 3(x + 2)
Perform the indicated operation.

1. $3.625 - 3.86$
2. $63.18 \times -.36$
3. $386.53 \div .04$

Perform the indicated operation.

4. \(\frac{16}{3} - \frac{5}{4}\)
5. \(\frac{16}{49} \times \frac{14}{8}\)
6. \(\frac{3}{88} \div \frac{21}{48}\)

Solve, convert or simplify.

7. $\sqrt{25}$
8. $5^6$
9. $4\sqrt{0996}$

10. $9.6 \times 10^8 - 6.38 \times 10^6$
11. $8.35 \times 10^7 \cdot 2.16 \times 10^5$

Convert to decimal notation (round to four decimal places) and then to a percent.

12. \(\frac{85}{100}\)
13. \(\frac{60}{75}\)
14. \(\frac{16}{24}\)
15. \(\frac{6}{10}\)
16. \(\frac{55}{80}\)
17. \(\frac{10}{86}\)

Follow order of operations to solve.

18. $18.25 + 16 \div 4 - 3.786$
19. $25 \div 5 + 36 \div 12 + 16.85$

Evaluate.

20. \(\frac{x+y}{5}, x = 23, y = 2\)
21. $A = \pi r^2, r = 4m$
22. $V = \frac{1}{3} \pi r^2 h, r = 7ft, h = 25ft$

Combine like terms and simplify.

23. $8a + 25a$
24. $21m^2 + 85 - 15m^2 + 16$
25. $x - 37y + 16x + 13y$

26. $2(5x + 6) + 3x$
27. $2(11x - 2a) + 27a - 3z$
28. $-\left(\frac{1}{3}a + \frac{2}{5}\right) + 2$

29. $2^3 - 5(3x + 8) - 10$
30. $23 + 5t + 7y - t - y - 27$

31. $2[(6 - 3(2x - 3)) - [2(-x + 1) - 3(-5)]]$
32. $-4[3(x - 2) + 7] - [4(3x + 2) + 3]]$
33. $7[2 - [3(11 - 2x) + 1] - 8(2x - 4)]$

Factor.

34. $10y + 5$
35. $16t - 24s$
36. $4a - 6b + 12c$
Answers
1. −.235
2. −22.7448
3. 9663.25
4. \( \frac{49}{12} \)
5. \( \frac{4}{7} \)
6. \( \frac{6}{77} \)
7. 5
8. 15625
9. 8
10. \( 9.5362 \times 10^8 \)
11. \( 1.8036 \times 10^{13} \)
12. .85, .85%
13. .8,80 %
14. .6667, 66.67 %
15. .6,60 %
16. .6875, 68.75%
17. .1163, 11.63%
18. 18.464
19. 24.85
20. 5
21. 50.27m²
22. 1282.82ft³
23. 33a
24. \( 6m^2 + 101 \)
25. 17x − 24y
26. 13x + 12
27. 23a + 19z
28. \( \frac{1}{3}a + \frac{8}{5} \)
29. −15x − 42
30. 4t + 6y − 4
31. −8x − 4
32. 36x + 40
33. −70x
34. 5(2y + 1)
35. 8(2t − 3s)
36. \( 2(2a − 3b + 6c) \)
# Chapter 1 Review 1

## 1.1 Arithmetic of Decimals, Positives and Negatives

<table>
<thead>
<tr>
<th><strong>Addition of Decimals</strong></th>
<th><strong>Subtraction of Decimals</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Line up decimals</td>
<td>1. Biggest on top</td>
</tr>
<tr>
<td>2. Add in columns</td>
<td>2. Line up decimals; subtract in columns</td>
</tr>
<tr>
<td>3. Carry by 10’s</td>
<td>3. Borrow by 10’s</td>
</tr>
<tr>
<td>4. Right size:</td>
<td>4. Strongest wins</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Multiplication of Decimals</strong></th>
<th><strong>Division of Decimals</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Multiply each place value</td>
<td>1. Set up</td>
</tr>
<tr>
<td>2. Carry by 10’s</td>
<td>2. Add zeros</td>
</tr>
<tr>
<td>3. Add</td>
<td>2. Divide into first</td>
</tr>
<tr>
<td>4. Right size:</td>
<td>3. Multiply</td>
</tr>
<tr>
<td></td>
<td>1. Add up zeros or decimals</td>
</tr>
<tr>
<td></td>
<td>2. Negatives</td>
</tr>
<tr>
<td></td>
<td>2. Borrow by denominator</td>
</tr>
<tr>
<td></td>
<td>3. Strongest wins</td>
</tr>
</tbody>
</table>

## 1.2 Arithmetic of Fractions, Positives and Negatives

<table>
<thead>
<tr>
<th><strong>Addition of Fractions</strong></th>
<th><strong>Subtraction of Fractions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Common denominator</td>
<td>1. Biggest on top</td>
</tr>
<tr>
<td>2. Add numerators</td>
<td>2. Common denominator:</td>
</tr>
<tr>
<td>3. Carry over denominator</td>
<td>Sub. numerators</td>
</tr>
<tr>
<td></td>
<td>2. Multiply the denominators</td>
</tr>
<tr>
<td></td>
<td>3. Prime factorization</td>
</tr>
<tr>
<td></td>
<td>1. Observation</td>
</tr>
<tr>
<td></td>
<td>2. Multiply numerators</td>
</tr>
<tr>
<td></td>
<td>3. Strongest wins</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Multiplication of Fractions</strong></th>
<th><strong>Division of Fractions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No common denominators</td>
<td>1. Improper Fractions</td>
</tr>
<tr>
<td>2. Multiply numerators</td>
<td>2. Keep it, change it, flip it</td>
</tr>
<tr>
<td>3. Multiply denominators</td>
<td>3. Multiply</td>
</tr>
</tbody>
</table>

**LCM:** Least common multiple, easily found by using prime factorization
## 1.3 Exponents, Roots, and Scientific Notation

<table>
<thead>
<tr>
<th>Solving Exponents</th>
<th>Converting to Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solving Roots</strong></td>
<td><strong>1.</strong> Decide which way the decimal will go&lt;br&gt;<strong>2.</strong> Count the number of times it moves&lt;br&gt;<strong>3.</strong> Multiply $\times 10$ to the number: exponent is&lt;br&gt;+ if to the right, - if to the left</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Addition/Subtraction of Scientific Notation</th>
<th>Multiplication with Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> Set it up&lt;br&gt;<strong>2.</strong> Add/Subtract Columns&lt;br&gt;<strong>3.</strong> Carry or borrow by 10’s&lt;br&gt;<strong>4.</strong> Strongest number wins&lt;br&gt;<strong>5.</strong> Put into scientific notation (left – up)</td>
<td><strong>1.</strong> Multiply Decimals&lt;br&gt;<strong>2.</strong> Add exponents&lt;br&gt;<strong>3.</strong> Put into scientific notation (left – up)</td>
</tr>
</tbody>
</table>

## 1.4 Percents

<table>
<thead>
<tr>
<th>Converting Percents</th>
<th>Solving Percent Problems with “of”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> If fraction, solve for decimals&lt;br&gt;<strong>2.</strong> Move decimal 2 places&lt;br&gt;<strong>3.</strong> “OF” means times</td>
<td><strong>1.</strong> Turn percent into a decimal&lt;br&gt;<strong>2.</strong> Multiply the two numbers together</td>
</tr>
</tbody>
</table>

## 1.5 Rounding and Estimation; Order of Operations

<table>
<thead>
<tr>
<th>Rounding</th>
<th>Getting the Right Order</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rounding</strong>&lt;br&gt;Rounding to the nearest thousand, hundred, ten, one, tenth, hundredth, thousandth, etc.</td>
<td><strong>P.E.M.D.A.S.</strong>&lt;br&gt;Parenthesis, Exponents, Multiplication/Division from left to right, Addition/Subtraction from left to right</td>
</tr>
<tr>
<td><strong>Estimation</strong>&lt;br&gt;Round the given values to make the problem easier, then do the easy problem</td>
<td></td>
</tr>
</tbody>
</table>
### 1.6 Variables and Formulas

**Shape Formulas**
Review the two pages of formulas for different properties of different shapes.

**Distance, Rate, and Time**  
\[ rt = d \]  
\( r = \text{rate}, \ t = \text{time}, \ d = \text{distance} \)

**Tax**  
\[ T = rP \]  
\( T = \text{tax}, \ r = \text{tax rate}, \ P = \text{original price} \)

**Discount**  
\[ D = rP \]  
\( D = \text{discount}, \ r = \text{discount rate}, \ P = \text{original price} \)

**Simple Interest**  
\[ I = Prt \]  
\( I = \text{interest}, \ P = \text{principal amount}, \ r = \text{interest rate}, \ t = \text{time} \)

**Fahrenheit to Celsius**  
\[ C = \frac{5}{9}(F - 32) \]  
\( F = \text{Fahrenheit deg.}, \ C = \text{Celsius deg.} \)

**Celsius to Fahrenheit**  
\[ F = \frac{9}{5}C + 32 \]  
\( F = \text{Fahrenheit deg.}, \ C = \text{Celsius deg.} \)

### 1.7 Laws of Simplifying

**Commutative Properties of Addition and Multiplication**
- A: \( 5 + 7 = 7 + 5 \)
- M: \( 3xy = y3x \)

**Associative Properties of Addition and Multiplication**
- A: \( 3 + (x + 8) = (3 + x) + 8 \) (just be careful with those negative signs!)
- M: \( 6 \cdot (4 \cdot x) = (6 \cdot 4) \cdot x \)

**Identity Properties of Addition and Multiplication**
- A: \( x + 0 = x \)
- M: \( x \cdot 1 = x \)

**Inverse Properties of Addition and Multiplication**
- A: \( 9x - 9x = 0 \)
- M: \( -8 \cdot -\frac{1}{8} = 1 \)

**Distributive Property**
- A&M: \( -3(2x + 1) = -6x - 3 \)

And, as always, don’t forget the order of operations!
1. Create a visual chart of all of the methods, formulas, and examples from studying how to solve these linear equations.

Perform the indicated operations.

2. \((-7) + (-4)\)
3. \(244 - (-326)\)
4. \(\frac{241.32}{-413.86}\)
5. \(-312.01 - 646.23\)
6. \(112.3 \times -12.1\)
7. \(3.6 \cdot (-1.4)\)
8. \((-4) \div .0002\)
9. \(24 \div (-.8)\)
10. \(.000248 \div .138\)

Change these fractions into decimals or these decimals into fractions.

11. \(\frac{27}{34}\)
12. \(\frac{15}{8}\)
13. \(\frac{87}{9}\)
14. \(.324\)

Perform the indicated operations.

15. \(\frac{-1}{3} + \frac{5}{6}\)
16. \(\frac{13}{23} + 2\)
17. \(\frac{9}{10} - \frac{4}{13}\)
18. \(\frac{8}{7} - \frac{4}{3}\)
19. \(\frac{5}{7} \cdot \frac{4}{3}\)
20. \(\frac{-24}{25} \div \frac{2}{5}\)

Write in exponential notation and evaluate.

21. \(7 \cdot 7 \cdot 7\)
22. \(-8 \cdot -8\)
23. \(2.2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)

Find the roots.

24. \(\sqrt[3]{81}\)
25. \(\sqrt[5]{-248832}\)
26. \(-\sqrt{169}\)

Convert between scientific notation and standard form.

27. \(6.43 \times 10^8\)
28. \(2.754 \times 10^{-5}\)
29. \(.0000000000789\)
30. \(1,234,000,000\)

Perform the indicated operations.

31. \(1.23 \times 10^4 + 7.89 \times 10^3\)
32. \(1.78 \times 10^{-8} - 2.65 \times 10^{-8}\)
33. \(2.4 \times 10^{-2} \cdot 3 \times 10^4\)

Evaluate and round to 3 decimal places.

34. \(38\%\) of 72
35. \(95\%\) of 643
36. \(50\%\) of 124

Follow the order of operations to solve.

37. \(3 + (-4) \div 2 - 6\)
38. \(4^2 + 3 \cdot (2 + 4) - 6 \times 7\)
39. \(34 \cdot 2 + 12 \div 2 + 52 - 12 \div 3\)
40. \(\left(\frac{3}{4} \cdot \frac{2}{7} - 4 + \frac{11}{14}\right) - 2^3\)
Write an equivalent expression.

41. \(-1 \geq -1.5\)  
42. \(140 < 196\)  
43. \(-1 > -8\)

Find the absolute value.

44. \(|-10|\)  
45. \(|17 - 7|\)  
46. \(|2(4 - 8)|\)

47. A large rug has a length of 6 ft, and its width is 1 ft less than twice the length. What is the area of the rug?

48. For dinner you bought 2 hamburgers for $1.99 each and some French fries for $1.39. How much total tax will be charged on your meal if the current tax rate is 7%?

49. The weather forecast in Brazil is 33° Celsius. How warm is that in degrees Fahrenheit?

Evaluate the expression with the given variable(s).

50. \(3.14 \cdot r^2 \cdot h; \text{ when } r = 3, h = 6\)  
51. \(\sqrt{a^2 + b^2}; \text{ when } a = 5, b = 12\)

Simplify by collecting like terms.

52. \(7y - 3(y - 4)\)  
53. \(a - 6(4 \cdot 2 + 3a)\)

54. \(3xy + 4(x - y) - x(-2 + y)\)  
55. \(\frac{2}{3}(3k + 6) - \frac{5}{2}(4p + 2k)\)

Factor.

56. \(7c - 14b\)  
57. \(12x + 28y - 16\)
Answers:
1. It better be good.
2. -11
3. 570
4. -172.54
5. -958.24
6. -1,358.83
7. -5.04
8. -20000
9. -30
10. 0.00180
11. 0.79412
12. 1.875
13. 9.66
14. \( \frac{81}{250} \)
15. \( \frac{7}{2} \)
16. \( \frac{59}{23} \)
17. \( \frac{27}{130} \)
18. \( \frac{52}{21} \)
19. \( \frac{20}{21} \)
20. \( -\frac{12}{5} \)
21. \( 7^4, 2401 \)
22. \( (-8)^3, -512 \)
23. \( 2^{11}, 2048 \)
24. 3
25. -12
26. -13
27. 643,000,000
28. 0.00002754
29. \( 7.89 \times 10^{-12} \)
30. \( 1.234 \times 10^9 \)
31. \( 2.019 \times 10^4 \)
32. \( -8.7 \times 10^{-9} \)
33. \( 7.2 \times 10^2 \)
34. 27.36
35. 610.85
36. 62
37. -5
38. -8
39. 122
40. -11
41. \(-1.5 \leq -1\)
42. 196 > 140
43. \(-8 < -1\)
44. 10
45. 10
46. 8
47. \(66 \text{ ft}^2\)
48. $0.38$
49. \(91.4^\circ\text{ F}\)
50. 169.56
51. 13
52. \(4y + 12\)
53. \(-17a - 48\)
54. \(2xy + 6x - 4y\)
55. \(-3k - 10p + 4\)
56. \(7(c - 2b)\)
57. \(4(3x + 7y - 4)\)
Perform the indicated operations.

1. \((-12) + 11\)
2. \((-18) - (-326)\)
3. \(12.613 - 731.165\)
4. \(-316.14 \times 123.5\)
5. \(-1.8 \cdot (-.21)\)
6. \((-1) \cdot 36\)
7. \((-76) ÷ 3\)
8. \(49 ÷ (-.82)\)
9. \(.04 ÷ .5\)

Change these fractions into decimals or these decimals into fractions.

10. \(\frac{7}{3}\)
11. \(\frac{1}{18}\)
12. \(\frac{47}{26}\)
13. \(.512\)

Perform the indicated operations.

14. \(\frac{2}{13} + 8\)
15. \(\frac{5}{12} + \frac{71}{18}\)
16. \(\frac{8}{3} - \frac{2}{15}\)
17. \(\frac{7}{90} - \frac{1}{15}\)
18. \(-\frac{6}{5} \cdot \frac{-5}{7}\)
19. \(-\frac{9}{16} ÷ -3\)

Write in exponential notation and evaluate.

20. \(9\cdot 9\cdot 9\)
21. \(-2\cdot -2\cdot -2\)
22. \(3\cdot 3\cdot 3\cdot 3\)

Find the roots.

23. \(\sqrt[3]{279936}\)
24. \(\sqrt{2197}\)
25. \(-\sqrt{441}\)

Convert between scientific notation and standard form.

26. \(7.96 \times 10^3\)
27. \(1.8 \times 10^{-7}\)
28. \(.00314\)
29. \(7,895,000\)

Perform the indicated operations.

30. \(4.12 \times 10^5 + 1.16 \times 10^5\)
31. \(2.3 \times 10^{-2} - 9.9 \times 10^{-4}\)
32. \(8.1 \times 10^{10} ÷ 5.3 \times 10^9\)

Evaluate.

33. \(12\% \text{ of } .14\)
34. \(88\% \text{ of } 888\)
35. \(151\% \text{ of } 113\)

Follow the order of operations to solve.

36. \(3 ÷ \frac{3}{4} + (7 - 2 \times 6)\)
37. \((3^4 - 27 ÷ 3) ÷ 4 + 4 \cdot (-3 \cdot 2) + 7\)
38. \[ 2 + 9 \cdot 6 - \frac{31}{3} - 8^2 \div 2^3 \]

Write an equivalent expression.

40. \(-1 \leq 5.9\)  
41. \(47 < 99\)  
42. \(-16 > -120\)

Find the absolute value.

43. \(|4|\)  
44. \(|5 - 7|\)  
45. \(|3(1 - 6)|\)

46. A circular swimming pool is covered by a tarp at night that exactly matches the size of the pool. The radius of the pool is 3m. What is the area of the tarp with the same radius?

47. One side of a triangular tent has a height of 4 ft and a base that is 3 less than twice the height. What is the area of this triangle?

48. A shirt is on sale for 15\% off the original price. If the original price was $15.95, how much is the discount?

Evaluate the expression with the given variable(s).

49. \[ \frac{4}{3} \pi r^3 \text{ when } r = 3 \]

Simplify by collecting like terms.

51. \[4t - 3(8 - t)\]

52. \[-12s + 7(4s + 3 \cdot 2)\]

53. \[-4ab + 2(a + 3b) - a(b - 7)\]

54. \[\frac{2}{5}(10x + 15) - \frac{5}{2}(4x + 2y)\]

Factor.

55. \[6x - 2y\]

56. \[20 + 5b + 15c\]
Answers:

1. -1
2. 308
3. -718.552
4. -39,043.29
5. .378
6. -36
7. -25.3
8. -59.756
9. .08
10. 2.3
11. 0.05
12. 1.808
13. \(\frac{64}{125}\)
14. \(\frac{106}{13}\)
15. \(\frac{157}{36}\)
16. \(\frac{38}{15}\)
17. \(\frac{1}{90}\)
18. \(\frac{6}{7}\)
19. \(\frac{3}{16}\)
20. \(9^4, 6561\)
21. \((-2)^4\) or \(2^4, 16\)
22. \(3^5, 243\)
23. 6
24. 13
25. -21
26. 7960
27. .00000018
28. \(3.14 \times 10^{-3}\)
29. \(7.895 \times 10^6\)
30. \(5.28 \times 10^5\)
31. \(2.201 \times 10^{-2}\)
32. 15.28 or \(1.528 \times 10^1\)
33. .0168
34. 781.44
35. 170.63
36. -1
37. 1
38. \(\frac{113}{3}\)
39. -14
40. \(5.9 \geq -1\)
41. \(99 > 47\)
42. \(-120 < -16\)
43. 4
44. 2
45. 15
46. \(28.27 \text{ m}^2\)
47. \(10 \text{ ft}^2\)
48. $2.39
49. 113.1
50. 5
51. \(7t - 24\)
52. \(16s + 42\)
53. \(-5ab + 9a + 6b\)
54. \(-6x - 5y + 6\)
55. \(2(3x - y)\)
56. \(5(4 + b + 3c)\)
Chapter 2: SOLVING: EQUATIONS & INEQUALITIES

OVERVIEW

The 3-Step Process to SOLVING:

2.1 Steps 2 & 3: The Addition and Multiplication Principles
2.2 Word Problems: Translation, Substitution, Shapes, Formulas
2.3 Step 1: Simplify the Equation: Parentheses, Like Terms
2.4 Word Problems: Percents
2.5 Step 1: Simplify the Equation: Fractions
2.6 Inequalities
2.7 Word Problems: Inequalities
SUCCESS STORIES from the field…

“My attitude is, if you push me towards something that you think is a weakness, then I will turn that perceived weakness into a strength.”

Michael Jordan

Some sports enthusiasts have touted Michael Jordan to be one of the greatest athletes of all time. Some of his achievements include 6 NBA championships, 5 time league MVP, two time U.S. Gold Medal winner, 10 NBA scoring titles, and all time career scoring average record. Many people are unaware that his path was not always easy. Did you know that he did not make his varsity high school team? His sophomore year at Laney high school, after missing the varsity cut, Michael began working on his game before school. In time he made the team, and eventually was named a High School All-American.

When Michael went to college at the University of North Carolina, did you know he was told he’d never start? At practice, he was the first one on the floor and the last one to leave. He was the only freshman on the court when he took the final shot of the NCAA championship game that won the title for UNC in 1982 with seconds left on the clock.

When Michael became a pro in the NBA, it was immediately apparent that he was a tremendous offensive power. He jumped, scored, and dunked like no one else. He led the league in scoring. However, critics said that was all he could do. He focused on his defensive game in the off-season and the next year again won the scoring title… along with the defensive player of the year award. He shares the record for the most seasons leading the league in steals to this day.

When people said he would never win a title, he won three. When he retired from the NBA to then return two years later, he was not in his peak physical shape. Critics again said he’d lost his step. Again in the off-season he worked relentlessly. The next season he led his team to the all-time record for wins in a season at 72, gathered three more championships, three more scoring titles, and two league MVP titles.

Michael has stated alongside all his basketball accomplishments and winning statistics, “I’ve missed more than 9000 shots in my career. I’ve lost almost 300 games. Twenty-six times, I’ve been trusted to take the game winning shot and missed. I’ve failed over and over and over again in my life. And that is why I succeed. If you do the work you get rewarded. There are no shortcuts in life.”
Section 2.1

**INTRODUCTION TO SOLVING AND THE 3-STEP PROCESS**

Remember in Chapter 1 in our introduction to Algebra, we talked about the two types of problems in algebra. Chapter 1 was about the simplify type. In this chapter we examine the solve type.

**ALGEBRA: Two Types of Problems**

**SIMPLIFY**
- CH.1
  - No “=” signs (or >,<, etc.)
  - Example: $2x + 3x$

**SOLVE**
- CH.2
  - Uses “=” signs (or >,<, etc.)
  - Example: $3x = 15$
  - Find out what $x$ equals

**EQUATION**: A mathematical sentence. It must have an equal sign and something (an expression) on each side. Examples: $1+3 = 4$; $x=5$; or $x+7 = 10$

**SOLUTION**: Any number when replaced for the variable that makes an equation true.
- Example: $3$ is the “SOLUTION” for the equation $x+7 = 10$

**SOLVE**: Find all the “SOLUTIONS” for an equation.

**EQUIVALENT EQUATIONS**: Equivalent is a big word for equal or SAME. Thus equivalent equations have the same solutions.

**SOLUTIONS of EQUATIONS**:
To determine if something is a SOLUTION to an EQUATION, we simply…

**Checking a Solution**
1. Plug the solution into the equation
2. Simplify the equation using the order of operations
3. If the result is a true statement, the answer is a solution
   - If the result is a false statement, the answer is not a solution

Follow the steps above in the examples that follow…
The GOLDEN Directions:
In this chapter, we learn to SOLVE Equations. The following process is the “GOLD” of this chapter.

3-Step Process to Solving
GOAL: Get $x$ alone ($x$ will represent any variable)
1. SIMPLIFY
   A) Get rid of fractions (multiply all by LCD)
   B) Distribute across ($\phantom{x}$)
   C) Combine “like terms” and get all $x$’s on the same side.
2. ADDITION PRINCIPLE
3. MULTIPLICATION PRINCIPLE

We will use this process in every chapter 2 section. We begin teaching the process in the next few pages. We’ll actually start with learning steps 2 & 3, and then in the next section we’ll cover step 1. This will take us on a gradual journey from the basics to more complex solving problems. To understand steps 2 & 3, we must learn the BALANCE RULE…

THE BALANCE RULE

Definition of ALGEBRA:
Math is the Manipulation of #'s to look different, yet remain the SAME.

One of the great tools of manipulation in SOLVE problems is the BALANCE RULE. It allows us to manipulate equations to look different yet remain the SAME (equivalent equations). This rule is illustrated in the following example…
This rule of balance is used in mathematical equations.

**Balance Rule of Solving:**
Whatever I do to one side of the equal sign, I must do the exact same to the other side to maintain equality.

Here’s a simple example of how the Balance Rule works in Arithmetic:

\[
\begin{align*}
23 &= 23 \\
2(23) &= 2(23) \\
46 &= 46 \\
46 - 4 &= 46 - 4 \\
42 &= 42 \\
42 \div 7 &= 42 \div 7 \\
6 &= 6
\end{align*}
\]

We start with \(23 = 23\). By multiplying each side by 2 we get \(46 = 46\). We then subtracted 4 from each side, and then divided each side by 7 to get \(6 = 6\).
Again we started with $23 = 23$. We manipulated each equation to look different, yet remain equal or the same. It was critical that we did the same thing to both sides each step of the way or we would have lost our balance or equality. For instance had we first multiplied one side by 2, but not the other we would have got $46 = 23$, which is not a true statement.

Use the BALANCE RULE to keep the equations below equivalent or the SAME.

| 3 | 36 = 36 | 4 | $x - 7 = 14$ | 5 | $\frac{12}{4} = \frac{12}{4}$ | 6 | $\frac{7x}{7} = \frac{21}{7}$ |
|---|---|---|---|---|---|---|
| $-4$ | $-4$ | $+7$ | $+7$ | $+7$ | $+7$ | $+7$ |

Now…to do that I must do the same thing (in blue) to the other side that was done to the 1st side.

<table>
<thead>
<tr>
<th>36 = 36</th>
<th>$x - 7 = 14$</th>
<th>$\frac{12}{4} = \frac{12}{4}$</th>
<th>$\frac{7x}{7} = \frac{21}{7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32 = x = 3 = x = 3$</td>
<td>$x = 21$</td>
<td>$x = 3$</td>
<td></td>
</tr>
</tbody>
</table>

C 3-STEP PROCESS TO SOLVING – STEP 2: The Addition Principle

The Addition Principle is inspired by the BALANCE RULE. Remember in SOLVE problems, the GOAL is to get $x$ alone. The Addition Principle (could aptly be named the Addition OR Subtraction Principle) states the following…

**Addition Principle:**

To Get $x$ alone…

1) Identify who is being added or subtracted to your variable
2) Do the opposite (operation) to both sides of the equation.

**EXAMPLES**

Let’s use the Addition Principle to first solve a simple equation we could do in our heads…just to prove it works. Follow the steps above. Then we’ll use it on a problem we may not be able to do in our heads. Let’s also check our answers using the steps we learned earlier in this chapter.

<table>
<thead>
<tr>
<th>7</th>
<th>$x + 5 = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5$</td>
<td>$-5$</td>
</tr>
</tbody>
</table>

Who is being added or subtracted from $x$? 5 (being added)

Do the opposite (subtract) to both sides.

$x + 0 = 10$

$x = 10$

**CHECK**

Is 10 the solution to the equation: $x + 5 = 15$?

$10 + 5 = 15$

$15 = 15 \checkmark$

True. 10 is a Solution
**8**

\[ a - 23 = -7 \]

Who is being added or subtracted from \( a \)? 23 (being subtracted)

Do the opposite (add) to both sides.

\[
\begin{align*}
+23 & \quad +23 \\
\hline
a + 0 & = 16 \\
\hline
a & = 16
\end{align*}
\]

**CHECK**

Is 16 the solution to the equation: \( a - 23 = -7 \)?

\[
\begin{align*}
16 - 23 & = -7 \\
-7 & = -7 \checkmark
\end{align*}
\]

True. 16 is a Solution

---

**9**

\[ -4.3 = y - 7.7 \]

Who is being added or subtracted from \( y \)? 7.7 (being subtracted)

Do the opposite (add) to both sides.

\[
\begin{align*}
+7.7 & \quad +7.7 \\
\hline
3.4 & = y + 0 \\
\hline
3.4 & = y
\end{align*}
\]

**CHECK**

Is 3.4 the solution to the equation: \( -4.3 = y - 7.7 \)?

\[
\begin{align*}
-4.3 & = 3.4 - 7.7 \\
-4.3 & = -4.3 \checkmark
\end{align*}
\]

True. 3.4 is a Solution

---

**10**

\[ -\frac{3}{4} + x = \frac{5}{8} \]

Who is being added or subtracted from \( x \)? \(-\frac{3}{4}\) (being added)

Do the opposite (subtract) to both sides. \(- \left(-\frac{3}{4}\right) = +\frac{3}{4}\)

\[
\begin{align*}
+\frac{3}{4} & \quad +\frac{3}{4} \\
\hline
0 + x & = \frac{11}{8} \\
\hline
x & = \frac{11}{8}
\end{align*}
\]

**CHECK**

Is \( \frac{11}{8} \) the solution to the equation: \( -\frac{3}{4} + x = \frac{5}{8} \)?

\[
\begin{align*}
-\frac{3}{4} + \frac{11}{8} & = \frac{5}{8} \\
\frac{5}{8} & = \frac{5}{8} \checkmark
\end{align*}
\]

True. \( \frac{11}{8} \) is a Solution
The Multiplication Principle is also inspired by the BALANCE RULE. Remember in SOLVE problems, the **GOAL is to get** \( x \) **alone**. The Multiplication Principle (could aptly be named the Multiplication OR Division Principle) states the following...

**Multiplication Principle:**

To Get \( x \) alone...

1) Identify who is being multiplied or divided to your variable
2) Do the opposite (operation) to both sides of the equation.

### EXAMPLES

Again, let’s use the Multiplication Principle to first solve a simple equation we could do in our heads…just to prove it works. Follow the steps above. Then we’ll use it on a problem we may not be able to do in our heads. Let’s also check our answers again.

**Example 11**

\[
3x = 15
\]

Who is being multiplied or divided from \( x \)? 3 (being mult.)

Do the opposite (divide) to both sides.

\[
\begin{align*}
\frac{3x}{3} &= \frac{15}{3} \\
x &= 5
\end{align*}
\]

**CHECK**

Is 5 the solution to the equation: \( 3x = 15 \)?

\[
\begin{align*}
3(5) &= 15 \\
15 &= 15 \checkmark
\end{align*}
\]

True. 5 is a Solution

**Example 12**

\[
\frac{x}{-4} = -12
\]

Who is being multiplied or divided from \( x \)? -4 (being divided)

Do the opposite (multiply) to both sides.

\[
\begin{align*}
-4 \left( \frac{x}{-4} \right) &= -4(-12) \\
x &= 48
\end{align*}
\]

**CHECK**

Is 48 the solution to the equation: \( \frac{x}{-4} = -12 \)?

\[
\begin{align*}
\frac{48}{-4} &= -12 \\
-12 &= -12 \checkmark
\end{align*}
\]

True. 48 is a Solution
This is a great example illustrating that when we solve…we want to know what \( x \) equals, not what \(-x\) equals…

\[-y = 14\]
\[-1y = 14\]
\[\frac{-1y}{-1} = \frac{14}{-1}\]
\[y = -14\]

Who is being multiplied or divided from \( y \)?
-1 (being multiplied)

Do the opposite (divide) to both sides.

CHECK

Is -14 the solution to the equation: \(-y = 14\)?

\[-(-14) = 14\]
\[14 = 14\]
True. -14 is a Solution

Who is being multiplied or divided from \( b \)? \(\frac{4}{15}\) (being multiplied)

\[\frac{15}{4} \cdot \frac{2}{5} = \frac{15}{4} \cdot \frac{4}{15}\]
\[\frac{3}{2} \cdot \frac{15}{4} = \frac{15}{4} \cdot \frac{4}{15}\]
\[\frac{3}{2} = b\]

Do the opposite (divide) to both sides. \(\frac{4}{15} \div \frac{4}{15} = \frac{4}{15} \cdot \frac{15}{4}\) (see R.2)

CHECK

Is \(\frac{3}{2}\) the solution to the equation: \(\frac{2}{5} = \frac{4}{15} b\)?

\[\frac{2}{5} = \frac{4}{15} \cdot \frac{3}{2}\]
\[\frac{2}{5} = \frac{2}{5}\]
True. \(\frac{3}{2}\) is a Solution
Now that we’ve learned the two balance rule principles in the 3-Step Process to Solving, let’s summarize these steps and use them to solve some problems incorporating them both.

**Summary of Addition & Multiplication Principles:**
1. Identify: Who is sharing sides with \( x \) ?
2. Identify: What are they doing to \( x \)? (add, subt, mult, or divide?)
3. Do the opposite (operation) to both sides of the equation.

**IMPORTANT NOTE:** It’s important to follow the order of the 3-Step Process by first applying the Addition Principle (remove who is being added and/or subtracted to \( x \)) then the Multiplication Principle (who is being multiplied or divided to \( x \)). This is critical so as not to create fractions.

**3-Step Process to Solving**

**GOAL:** Get \( x \) alone (\( x \) will represent any variable)

1. **SIMPLIFY**
   - A) Get rid of Fractions (multiply all by LCD)
   - B) Distribute ( )
   - C) Combine Like Terms (L.T.) and Get all \( x \)’s to 1 side.

2. **ADDITION PRINCIPLE**

3. **MULTIPLICATION PRINCIPLE**

**EXAMPLES**

**SOLVE and CHECK using Steps 2 & 3 of the 3-Step Process to Solving:**

\[3x + 10 = 22\]

\[
\begin{align*}
-10 & \quad -10 \\
3x & \quad = 12 \\
\underline{3x} & \quad \underline{= 12} \\
\frac{3}{3} & \quad \frac{x}{3} \\
x & \quad = 4
\end{align*}
\]

**3-Step Process to Solving**

Who is sharing sides with \( x \)? \( \{10 \text{ & 3}\}

Who is being added/subtracted? \( \{10 \text{ (added)}\}

Do the opposite (subtract) to both sides.

Who is being multiplied/divided? \( \{3 \text{ (multiplied)}\}

Do the opposite (divide) to both sides.

**CHECK**

Is 4 the solution to the equation: \(3x + 10 = 22\)?

\[
\begin{align*}
3(4) + 10 & \quad = 22 \\
12 + 10 & \quad = 22 \\
22 & \quad = 22
\end{align*}
\]

True. 4 is a Solution
-2x – 7 = 14

\[ \begin{align*}
+7 &+7 \\
-2x &+ 21 \\
\end{align*} \]

\[ -2x = 21 \]

\[ \begin{align*}
\frac{-2x}{-2} &+ \frac{21}{-2} \\
x &+ \frac{-21}{2} \\
\end{align*} \]

\[ x = \frac{-21}{2} \]

**CHECK**

Is \( \frac{-21}{2} \) the solution to the equation:

\[ -2x – 7 = 14? \]

\[ -2 \left( \frac{-21}{2} \right) – 7 = 14 \]

\[ 21 – 7 = 14 \]

\[ 14 = 14 \]

True. \( \frac{-21}{2} \) is a Solution

**3-Step Process to Solving**

\[ 35 – y = 23 \]

\[ -35 \quad -35 \]

\[ -y = -12 \]

\[ \frac{-y}{-1} = \frac{-12}{-1} \]

\[ y = 12 \]

**CHECK**

Is 12 the solution to the equation:

\[ 35 – y = 23 \]

\[ 35 – 12 = 23 \]

\[ 23 = 23 \]

True. 12 is a Solution

\[ 12.4 – 4.5b = -16.4 \]

\[ -12.4 \quad -12.4 \]

\[ -4.5b = -28.8 \]

\[ \begin{align*}
-4.5b &+ -28.8 \\
\frac{-4.5b}{-4.5} &+ \frac{-28.8}{-4.5} \\
b &+ 6.4 \\
\end{align*} \]

**CHECK**

Is 6.4 the solution to the equation:

\[ 12.4 – 4.5b = -16.4? \]

\[ 12.4 – 4.5(6.4) = -16.4 \]

\[ 12.4 – 28.8 = -16.4 \]

\[ -16.4 = -16.4 \]

True. 6.4 is a Solution
2.1 EXERCISE SET

Find the Volume of a rectangular solid when the width, height and length are given.
Formula is \( V = lwh \)

1. \( l = 4 \text{ in} \)
   \( w = 2.5 \text{ in} \)
   \( h = 3 \text{ in} \)
   \( V = \)

2. \( l = 7 \text{ ft} \)
   \( w = 4 \text{ ft} \)
   \( h = 2.8 \text{ ft} \)
   \( V = \)

3. \( l = 7.2 \text{ m} \)
   \( w = 9 \text{ m} \)
   \( h = 3 \text{ m} \)
   \( V = \)

Find the Area of a trapezoid when the bases and height are given.
Formula is \( A = \frac{h(B+b)}{2} \)

4. \( B = 15 \)
   \( b = 10 \)
   \( h = 7 \)
   \( A = \)

5. \( B = 21 \)
   \( b = 11 \)
   \( h = 3 \)
   \( A = \)

6. \( B = 19 \)
   \( b = 6 \)
   \( h = 10 \)
   \( A = \)

Identify the property that is illustrated by each statement.

7. \( (8 + 5) + 3 = 3 + (8 + 5) \)
8. \( (3xy)7x = (3yx)7x \)
9. \( (8ab)7c = 8(ab7)c \)

Simplify.

10. \( 2s(t - 7) - 6t(s + 3) \)
11. \( 3(x^2 - 5n) + 3n - 7x^2 \)
12. \( 6kj - 7k + 8kj + 11 \)

Check to see if the specified number is a solution for the given equation.

13. \( y + 24 = 37 \)
   Is 13 a solution for \( y + 24 = 37 \)?

14. \( p + 14 = 32 \)
   Is 19 a solution for \( p + 14 = 32 \)?

15. \( 24; t - 34 = 58 \)

16. \( 45; x - 21 = 24 \)

Solve.

17. \( x + 4 = 13 \)

18. \( 13 + t = 27 \)

19. \( y + 17 = -12 \)

20. \( y + \frac{2}{7} = 6 \)

21. \( x + \frac{9}{2} = 4 \)

22. \( 8 = x - \frac{5}{8} \)

23. \( p - 16.2 = 11.2 \)

24. \( -6.1 + x = -6.7 \)

25. \( -4.2 + z = -3.1 \)

Solve.

26. \( - y = 15 \)

27. \( 45 = - x \)

28. \( - p = - 34 \)

29. \( \frac{8}{3}y = 16 \)

30. \( - \frac{x}{4} = \frac{1}{6} \)

31. \( \frac{7}{4} = - \frac{x}{5} \)

32. \( \frac{4}{5}p = - 5.6 \)

33. \( - \frac{4}{3}z = - 15.3 \)

34. \( - \frac{x}{14} = 6 \)
Use both the addition and multiplication principles together to solve the following.

2.1e

35. \(12x + 7 = 31\)  
36. \(4y + 18 = 30\)  
37. \(5z + 21 = 56\)

38. \(5x - 5 = 20\)  
39. \(3y - 7 = 27\)  
40. \(-8x - 10 = 62\)

41. \(-4x - 12 = 18\)  
42. \(2.7m + 12.13 = 20.5\)  
43. \(-3.5x + 2.4 = 24.1\)

Preparation: Read some of 2.2 and then

44. Solve for \(p\):
   \[3p + 7 = 15\]

45. Solve for \(p\):
   \[mp + t = q\]
Section 2.1

Answers
1. 30 in³
2. 78.4 ft³
3. 194.4 m³
4. 87.5
5. 48
6. 125
7. Commutative property of addition
8. Commutative property of multiplication
9. Associative property of multiplication
10. $-4st - 14s - 18t$
11. $-4x^2 - 12n$
12. $14kj - 7k + 11$
13. Yes
14. No
15. No
16. Yes
17. $x = 9$
18. $t = 14$
19. $y = -29$
20. $y = 5 \frac{5}{7} \text{ or } 4\frac{6}{7}$
21. $x = -\frac{1}{2} \text{ or } -0.5$
22. $x = 8\frac{5}{8} \text{ or } 6\frac{6}{8} \text{ or } 8.625$
23. $p = 27.4$
24. $x = -0.6$
25. $z = 1.1$
26. $y = -15$
27. $x = -45$
28. $p = 34$
29. $y = 6$
30. $x = -\frac{2}{3}$
31. $x = -\frac{35}{4} \text{ or } -8\frac{3}{4}$
32. $p = -7$
33. $z = 11.475$
34. $x = -84$
35. $x = 2$
36. $y = 3$
37. $z = 7$
38. $x = 5$
39. $y = \frac{24}{3} \text{ or } 11\frac{1}{3}$
40. $x = -9$
41. $x = -7.5 \text{ or } -15\frac{1}{2}$
42. $m = 3.1$
43. $x = -6.2$
44. In class
45. In class.
“When am I ever going to use this?” “Where would this be applicable?” All the way through math, students ask questions like these. Well, to the relief of some and the dismay of others, you have now reached the point where you will be able to do some problems that have been made out of real life situations. Most commonly, these are called, “story problems”.

The four main points to solving story problems are:

**D** - **Data.** Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

**V** - **Variable.** In all of these story problems, there is something that you don’t know, that you would like to. Pick any letter of the alphabet to represent this.

**P** - **Plan.** Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. This section of the book is divided up so as to explain most of the different kinds of patterns.

**E** - **Equation.** Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

You have been working with linear equations, which are equations in which the variables are only raised to the power of one (i.e. there are no exponents on the variables). Linear equations are found throughout mathematics and the real world. Here is a small outline of some applications of linear equations. You will be able to solve any of these problems by the same methods that you have just mastered.
I. Translation
The first application is when you simply translate from English into math.

EXAMPLES

1. Seven less than 3 times what number is 41?

7, 3, 41 are the numbers involved.
Let m be the number we don’t know.

Seven less than 3 times what number is 41?  

\[ 3m - 7 = 41 \]

P – Plan (We are translating)
E – Equation

Solve as before

\[ m = 16 \]

Answer: 16

2. Stacey traveled 141 miles while going 27 mph. Using the formula \( rt = d \) determine the time that she traveled.

\[ d = 141 \text{ miles} \]
\[ r = 27 \text{ mph} \]

V – Variable
P – Plan (Use formula given in problem)

E – Equation
Solve as before

\[ t = 3 \text{ hours} \]

Answer: 3 hours

II. Shapes
With many of the problems that you will have, pictures and shapes will play a very important role. When you encounter problems that use rectangles, triangles, circles or any other shape, I would suggest a few things:

1. Read the problem
2. READ the problem again.
3. READ THE PROBLEM one more time.

Once you draw a picture to model the problem, read the problem again to make sure that your picture fits.

Here are some formulas for common shapes that you will encounter. You should start to become familiar with them:
## Shape formulas:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>( P = 2l + 2w ) ( A = lw )</td>
<td>( P ) is the perimeter ( l ) is the length ( w ) is the width ( A ) is the Area</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>( P = 2a + 2b ) ( A = bh )</td>
<td>( P ) is the perimeter ( a ) is a side length ( b ) is the other side length ( h ) is height ( A ) is the Area</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>( P = b + a + B + d ) ( A = \frac{1}{2} h (B + b) )</td>
<td>( P ) is perimeter ( b ) is the little base ( B ) is the big base ( a ) is a leg ( h ) is height ( d ) is a leg ( A ) is the Area</td>
</tr>
<tr>
<td>Triangle</td>
<td>( P = s_1 + s_2 + s_3 ) ( A = \frac{1}{2} bh )</td>
<td>( P ) is the perimeter ( h ) is height ( b ) is base ( A ) is the Area</td>
</tr>
<tr>
<td>Triangle</td>
<td>( a + b + c = 180 )</td>
<td>( a ) is one angle ( b ) is another angle ( c ) is another angle</td>
</tr>
<tr>
<td>Rectangular Solid</td>
<td>( SA = 2lw + 2wh + 2lh ) ( V = lwh )</td>
<td>( l ) is the length ( h ) is the height ( w ) is the width ( SA ) is the Surface Area ( V ) is volume</td>
</tr>
<tr>
<td>Shape</td>
<td>Formulas</td>
<td>Notes</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| Circle   | $C = 2\pi r$  \quad $A = \pi r^2$         | $C$ is the Circumference or perimeter  
\quad $\pi$ is a number, about 3.14159 . . .  
\quad it has a button on your calculator  
\quad $r$ is the radius of the circle  
\quad $A$ is the area inside the circle. |
| Cylinder | $LSA = 2\pi rh$  \quad $SA = 2\pi rh + 2\pi r^2$ \quad $V = \pi r^2 h$ | $LSA$ is Lateral Surface  
\quad Area=Area just on the sides  
\quad $h$ is the height  
\quad $SA$ is total surface area  
\quad $\pi$ is a number, about 3.14159 . . .  
\quad it has a button on your calculator  
\quad $r$ is the radius of the circle  
\quad $V$ is Volume |
| Cone     | $LSA = \pi rl$  \quad $SA = \pi r^2 + \pi rl$ \quad $V = \frac{1}{3}\pi r^2 h$ | $h$ is the height  
\quad $r$ is the radius of the circle  
\quad $l$ is the slant height  
\quad $\pi$ is a number, about 3.14159 . . .  
\quad it has a button on your calculator  
\quad $SA$ is total surface area  
\quad $LSA$ is Lateral Surface  
\quad Area=Area just on the sides  
\quad $V$ is Volume |
| Sphere   | $SA = 4\pi r^2$  \quad $V = \frac{4}{3}\pi r^3$ | $r$ is the radius  
\quad $SA$ is the surface area  
\quad $V$ is the Volume |
If the perimeter of a rectangle is 56 in and the width is 12 in, what is the length?

\[ 56 \text{ in} = P \]
\[ 12 \text{ in} = w \]

\( L = \text{length we are trying to find} \)

\[ P = 2l + 2w \]

\[ 56 = 2l + 2(12) \]
\[ 56 = 2l + 24 \]

\[ 56 = 2l + 24 \]
\[ 18 = 2l \]
\[ 9 = l \]

Answer: length of rectangle is 9 in.

Using the formulas for a cylinder find the missing variable:

\[ r = 9 \text{ cm} \]
\[ h = ? \]
\[ V = 356 \text{ cm}^3 \]

\[ r = 9 \text{ cm} \]
\[ V = 356 \text{ cm}^3 \]
\( h = \text{height that we are trying to find} \)

\[ V = \pi r^2 h \]
\[ 356 = \pi (81) h \]
\[ 356 = 254.47h \]
\[ \frac{356}{254.47} = \frac{254.47h}{254.47} \]
\[ 1.4 = h \]

Answer: height is 1.4 cm.
III. Solving for a variable:

When given a formula, it is sometimes requested that you solve that formula for a specific variable. That simply means that you are to get that variable by itself. We use the same principles as in the previous section:

**EXAMPLES**

5. Solve \( rt = d \) for \( t \)

\[
rt = d \quad \text{for} \quad t \\
\frac{rt}{r} = \frac{d}{r} \\
t = \frac{d}{r}
\]

**Answer:** \( t = \frac{d}{r} \)

6. Solve \( y = bx + c \) for \( x \)

\[
y = bx + c \quad \text{for} \quad x \\
y - c = bx \quad \text{subtract “c” from both sides} \\
\frac{y - c}{b} = \frac{bx}{b} \\
\frac{y - c}{b} = x
\]

**Answer:** \( x = \frac{y-c}{b} \)

7. Solve \(-3m - 4pt = 7\) for \( m \)

\[
-3m - 4pt = 7 \quad \text{for} \quad m \\
-3m = 7 + 4pt \quad \text{add (4pt) to both sides} \\
\frac{-3m}{-3} = \frac{7 + 4pt}{-3} \\
m = \frac{7 + 4pt}{-3}
\]

**Answer:** \( m = \frac{7 + 4pt}{-3} \)
2.2 EXERCISE SET

Check to see if the specified number is a solution for the given equation.

2.1
1. \( 4; 7y + 13 = 15 \)
2. \( 9; 99 - 3p = 72 \)
3. \(-21; y + 4 = y + 4 \)

Solve.

4. \( t - 15 = 43 \)
5. \( y - 22 = 23 \)
6. \( p - 12 = -21 \)

7. \( \frac{8}{9} + y = \frac{13}{6} \)
8. \( 8.1 = 4.2 + x \)
9. \( 12.6 = z - 13.3 \)

10. \( \frac{2}{5}x = \frac{1}{10} \)
11. \( -\frac{4}{9}y = \frac{1}{3} \)
12. \( -\frac{5}{7} = -\frac{15}{14}z \)

13. \( 3.6y = 18 \)
14. \( 94.8 = 23.7x \)
15. \(-2.1z = 12.6 \)

16. \( 7y + 7 = 35 \)
17. \( 2z + 13 = 3 \)
18. \( 4y + 25 = 13 \)

19. \( 21 - x = 13 \)
20. \( 9 - 5y = 27 \)
21. \( -14 - 6y = 17 \)

2.2
22. 27 is 6 more than 3 times a number. What is the number?
23. 18 less than 5 times a number is 52. What is the number?

24. Using the formula \( I = Prt \) for interest, find the missing variable.
   \[ I = \$376.52 \]
   \( P = ? \)
   \( r = .06 \)
   \( t = 5 \)

25. If a cone has a Lateral Surface Area of 250 ft\(^2\), a radius of 8ft, what is the slant height of the cone?
26. If a cylinder has a volume of 538 cm\(^3\) and a radius of 6 cm, how tall is it?

27. Find the missing variable for a rectangle:
   \( P = 39 \text{ ft} \)
   \( w = 7.2 \text{ ft} \)
   \( l = \)

28. Find the missing variable for a cylinder:
   \( SA = 800 \text{ in}^2 \)
   \( h = \)
   \( r = 9 \text{ in} \)
Solve for the specified variable.

29. \( y = mx + b \) for \( b \)
30. \( 5m - 7 = r \) for \( m \)
31. \( A = 2\pi rh \) for \( h \)
32. \( A = \frac{1}{2}bh \) for \( b \)
33. \( 3m - 8qt = 14 \) for \( m \)
34. \( 19 = 3pqr \) for \( r \)
35. \( C = \frac{5}{9}(F - 32) \) for \( F \)
36. \( V = \frac{1}{3}\pi r^2h \) for \( h \)

Preparation.

37. After reading some from Section 2.3, try to solve this equation for \( x \).
\[ x + \underline{9} = 9 + \underline{9} \]

38. Solve the following for \( x \):
\[ 5x + 9y + 10p = 9y + 15 + 10p \]
Answers
1. No
2. Yes
3. Yes
4. \( t = 58 \)
5. \( y = 45 \)
6. \( p = -9 \)
7. \( y = \frac{23}{18} \)
8. \( x = 3.9 \)
9. \( z = 25.9 \)
10. \( x = \frac{1}{4} \)
11. \( y = -\frac{3}{4} \)
12. \( z = \frac{2}{3} \)
13. \( y = 5 \)
14. \( x = 4 \)
15. \( z = -6 \)
16. \( y = 4 \)
17. \( z = -5 \)
18. \( y = -3 \)
19. \( x = 8 \)
20. \( y = -\frac{18}{5} \)
21. \( y = -\frac{31}{6} \) or \(-5\frac{1}{6} \) or \(-5.16 \)
22. 7
23. 14
24. \$1255.07
25. 9.95 \( ft \)
26. 4.76 \( cm \)
27. 12.3 \( ft \)
28. 5.15 \( in \)
29. \( b = y - mx \)
30. \( m = \frac{r+7}{5} \)
31. \( h = \frac{A}{2\pi r} \)
32. \( b = \frac{2A}{h} \)
33. \( m = \frac{14+8qt}{3} \)
34. \( r = \frac{19}{3pq} \)
35. \( F = \frac{9}{5}C + 32 \)
36. \( h = \frac{3V}{\pi r^2} \)
37. In class.
38. In class.
Recall from section 2.1 that we are learning how to solve equations. We will now learn step 1 of the process.

**3-Step Process to Solving**

**GOAL:** Get $x$ alone ($x$ will represent any variable)

1. **SIMPLIFY**
   
   A) Distribute across $(\quad)$
   
   B) Get rid of fractions (multiply all by LCD)
   
   C) Combine “like terms” and get all $x$’s on the same side.

2. **ADDITION PRINCIPLE**

3. **MULTIPLICATION PRINCIPLE**

**EXAMPLES**

1. **Solve** $9(x + 2) - 4x = 28 + x + 2$

   First solve the parenthesis like we already learned how to do

   Combine all of the numbers with $x$’s on each side of the equation

   Get all of the $x$’s on one side of the equation and all of the numbers on the other by adding or subtracting

   Divide by the number attached to $x$ on both sides of the equation to get the $x$ alone

   **Answer:** $x = 3$
Solve \(24 - 2(3x - 4) = -4\)

First distribute into the parenthesis

\[24 - 2(3x - 4) = -4\]
\[24 - 6x + 8 = -4\]

Combine all of the like terms on each side of the equation

\(24 - 6x + 8 = -4\)
\(32 - 6x = -4\)
\((-32) \quad (-32)\)
\(-6x = -36\)
\(-6x = -36\)
\(\div (-6) \quad \div (-6)\)
\[x = 6\]

Answer: \(x = 6\)

In Chapter 1, Section 5, we discussed Order of Operations, and we said that you needed to work parenthesis first. The reason was that they needed to be taken care of before you move on. Well, in simplifying equations to solve, do the same thing – work across the parenthesis first – not just inside them but distributing into them:

**LAWS & PROCESSES**

Would you agree that: \(3 \cdot 4 + 3 \cdot 7 = 3 \cdot (4 + 7)\) ?

Check: \(12 + 21 = 33\)  \(\checkmark\) \(3 \cdot (11) = 33\)

What about this: \(4 \cdot x + 4 \cdot 6 = 4(x + 6)\) ?

They both end up being \(4x + 24\). What needs to be understood is how this principle works, and then to know that it is often necessary in simplifying equations.

**EXAMPLES**

Simplify: \(4[6(1 + x) - 3x] = 6 - 2(5 - x)\)

\(4[6(1 + x) - 3x] = 6 - 2(5 - x)\) Distribute into the parenthesis
\(4[6 + 6x - 3x] = 6 - 10 + 2x\) Distribute into the brackets
\(24 + 24x - 12x = 6 - 10 + 2x\) Combine like terms
\(24 + 12x = -4 + 2x\) Get all of the x’s on one side
\((-24)(-2x) \quad (-24) \quad (-2x)\)
\(10x = -28\) Divide by the number attached to the x
\(\div 10\) \(\div 10\)
\[x = 2.8\]

Answer: \(x = 2.8\)
We can see that there wouldn’t have been hope to solve that last example if we didn’t distribute across the parenthesis.

Now we reach a point where we should feel powerful. Remember that we can **add, subtract, multiply or divide** anything we want! (As long as we do it to both sides).

Particularly, if we don’t like the way that $11x$ is on the left hand side, we can **choose** to get rid of it! So, we subtract $11x$ from both sides of the equation:

\[
\begin{align*}
5x - 8 &= 11x - 56 \\
-11x &= -11x
\end{align*}
\]

Upon combining the like-terms, we get

\[
-6x - 8 = -56
\]

which now is able to be un-done easily:

\[
\begin{align*}
-6x &= -48 \\ 
x &= 8
\end{align*}
\]

**Special cases:**

What about $2x + 1 = 2x + 1$? Well if we want to get the x’s together we had better get rid of the $2x$ on one side. So we subtract $2x$ from both sides like this:

\[
\begin{align*}
2x + 1 &= 2x + 1 \\
-2x &= -2x \\
1 &= 1
\end{align*}
\]

The x’s all vanished!

This statement is **always** true no matter what x is. That is the point. x can be any number it wants to be and the statement will be true. **All numbers are solutions** to this equation.

On the other hand try to solve $2x + 1 = 2x - 5$

\[
\begin{align*}
2x + 1 &= 2x - 5 \\
-2x &= -2x \\
1 &= -5
\end{align*}
\]

Again, the x’s all vanished. This time it left an equation that is **never** true. No matter what x we stick in, we will never get 1 to equal -5. It simply will never work. **No solution.**
2.3 EXERCISE SET

Solve.

1. \( \frac{x}{-6} = -3 \)
2. \( 13.7 - 3.4t = -18.9 \)
3. \( -17 - 7m = -18 \)

4. \( \frac{3}{7}t + 1 = -11 \)
5. \( 9 = 3x + 17 \)
6. \( \frac{5x + 7}{4} = 13 \)

7. \( 8t + 3t + 14t - 17 = -17 \)
8. \( 94.8 = 23.7x - 13.5 \)

Solve for the specified variable.

9. \( p = fx + bn \quad \text{for } f \)
10. \( F = \frac{xf - xz}{2} \quad \text{for } f \)

11. \( M = 5t - 3p \quad \text{for } t \)
12. \( LSA = \pi rl \quad \text{for } r \)

13. \( E = Q - \frac{r_1}{r_2} \quad \text{for } Q \)
14. \( \frac{3s - 4g}{7} = c \quad \text{for } g \)

15. 48 is 9 more than 3 times a number. What is the number?
16. 18 less than 7 times a number is 80. What is the number?
17. If two angles of a triangle are 70° and 48°, what is the measure of the third angle?
18. What is the width of a rectangle that has an area of 390in² and a length of 20in?

19. Find the area of the shaded region:

![Diagram of a shaded region with dimensions 3cm x 8cm and a rectangle of 14 cm side]

20. What is the slant height of a cone that has radius of 7m and a surface area of 700m²?

21. What is the width of a rectangular solid that has a volume of 238mm³, a length of 17mm and a height of 2mm?

22. If a cone has a volume of 338 cm³ and a radius of 6 cm, how tall is it?

23. Find the missing variable for a parallelogram:
   \[ A = 64 \text{ in}^2 \]
   \[ h = \]
   \[ b = 12.6 \text{ in} \]
Solve.

24. \(5p + 12 = 33 - p\)  
25. \(7n + 18 = 5(n - 2)\)  
26. \(5x - 10 = 5x + 7\)

27. \(x - 7 = 15x\)  
28. \(2x - 4(x - 3) = -2x + 12\)  
29. \(.07x = 13 - .12x\)

30. \(.7(3x - 2) = 3.5x + 1\)  
31. \(.3x - 9 + 2x = 4x - 3\)  
32. \(.4y = 78 + .4y\)

33. \(7(x - 5) - 3x = 4x - 35\)  
34. \(9x - 4(x - 3) = 15x\)  
35. \(2x - 3x + 7x = 9x + 8x\)

Preparation.

36. Find the final price of an object that is $200 but has 15% off.

37. Find the final amount of a savings account that has $170 and then has 15% interest added to it.

38. After reading some of 2.4, try to find out what the original price of an object was if the final price after 15% off was $85.
Answers

1. \( x = 18 \)
2. \( t = 9.59 \)
3. \( m = \frac{1}{7} \)
4. \( t = -28 \)
5. \( x = -\frac{8}{3} \)
6. \( x = 9 \)
7. \( t = 0 \)
8. \( x = 4.57 \)
9. \( f = \frac{p-bn}{x} \)
10. \( f = \frac{2F+xz}{x} \)
11. \( t = \frac{M+3p}{5} \)
12. \( r = \frac{LSA}{\pi t} \)
13. \( Q = E + \frac{T_1}{T_2} \)
14. \( g = \frac{7c-3s}{-4} \) or \( \frac{3s-7c}{4} \)
15. \( x = 13 \)
16. \( x = 14 \)
17. \( 62^\circ \)
18. \( 19.5 \text{ in} \)
19. \( 129.9 \text{ cm}^2 \)
20. \( 24.83 \text{ m} \)
21. \( 7 \text{ mm} \)
22. \( h = 8.97 \text{ cm} \)
23. \( h = 5.08 \text{ in} \)
24. \( p = \frac{7}{2} \) or \( 3.5 \)
25. \( n = -14 \)
26. No solution
27. \( x = -\frac{1}{2} \)
28. All real numbers
29. \( 68.42 \)
30. \( x = -\frac{12}{7} \) or \( -1.71 \)
31. \( x = -3.53 \)
32. No solution
33. All real numbers
34. \( x = \frac{6}{5} \) or \( 1.2 \)
35. \( x = 0 \)
36. In class.
37. In class.
38. In class.
Substitution

Sometimes you are given a couple of different things to find instead of just one. In these cases, you can use the information you are given to substitute something in for one of the unknown variables.

EXAMPLES

1. Two numbers add to 15, and the second is 7 bigger than the first. What are the two numbers?

   \[ f + s = 15 \quad s = f + 7 \]

   D – add to 15; one 7 bigger than other
   V – f for first s for second

   \[ f + s = 15 \quad s = f + 7 \]
   \[ f + f + 7 = 15 \]
   \[ 2f + 7 = 15 \]
   \[ 2f = 8 \]
   \[ f = 4 \]

   S – Substitution. Substitute \( f + 7 \) in for \( s \)
   E – Write the equation to solve

   Answer: first number is 4, second number must be 11.

2. A man cuts a 65 inch board so that one piece is four times bigger than the other. What are the lengths of the two pieces?

   \[ x + y = 65 \quad y = 4x \]

   D – 65 inches total; one 4 times the other
   V – x for first y for second

   \[ x + y = 65 \quad y = 4x \]
   \[ x + 4x = 65 \]
   \[ 5x = 65 \]
   \[ x = 13 \]

   S – Substitution. Substitute \( 4x \) in for \( y \)
   E – Write the equation to solve

   Answer: first piece is 13in, second piece is 52in.
If a rectangle’s length is 5 more than 3 times the width and the perimeter is 58 mm, what are the dimensions of the rectangle?

\[2w + 2l = 58 \quad l = 3w + 5\]

D – 58 millimeters total; length 3 times the width plus 5
V – w for width, l for length

P – Substitution. Substitute 3w + 5 in for l

\[x + 4x = 65\]  \[x + 4x = 65\]  \[5x = 65\]  \[x = 13\]

E – Write the equation to solve
Solve the equation

Answer: first piece is 13 in, second piece is 52 in.

I have created a triangular garden such that the largest side is 8 m less than twice the smallest and the medium side is 12 m larger than the smallest side. If the total perimeter of the garden is 104 m, what are the lengths of the three sides?

\[s + m + l = 104 \quad l = 2s - 8 \quad m = s + 12\]

D – 104 m total
V – l for largest, m for medium, s for smallest
P – Substitution. Substitute f + 7 in for s

E – Write the equation to solve
Solve the equation

\[4s + 4 = 104\]
\[4s = 100\]
\[s = 25\]

Answer: smallest is 25 m, medium is 37 m, largest is 42 m.

We will now expand our word problems to dealing with percents, in very similar ways that we did problems in Chapter 1, Section 4.

B SIMPLE PERCENT PROBLEMS

DEFINITIONS & BASICS

Remember from earlier, if we break up the word “percent” we get “per” which means divide and “cent” which means 100. Notice that .72 is really the fraction \(\frac{72}{100}\). We see that when we write it as a percent instead of its numerical value, we move the decimal 2 places. Here are some more examples:

\[.73 = 73\%\]
\[.2 = 20\%\]
\[1 = 100\%\]
\[2.3 = 230\%\]
\[2.14 = 214\%\]

Section 2.4
The next reminder, before we start doing problems, is that the word “of” often means “times”. It will be especially true as we do examples like:

What is 52% of 1358?  
All we need to do is multiply (.52)(1358) which is 706.16

Sometimes however, it isn’t quite that easy to see what needs to be done. Here are three examples that look similar but are done very differently. Remember “what” means “x”, “is” means “=” and “of” means “times”.

What is 15% of 243?  
\[x = .15(243)\]

15 is what percent of 243?  
\[15 = x (243)\]

15 is 243% of what?  
\[15 = 2.43x\]

**EXAMPLES**

**5**

What is 26% of $40$?  
Set up an equation. Remember “what” means “x”, “is” means “=” and “of” means “times”

\[x = .26 \times 40\]
\[x = .26 \times 40\]
\[x = 10.4\]

Answer: 10.4

**6**

118.08 is what percent of 246?  
Set up an equation. Remember “what” means “x”, “is” means “=” and “of” means “times”

\[118.08 = x \times 246\]
\[118.08 = x \times 246\]
\[\div 246 \div 246\]
\[.48 = x\]
\[x = .48\]

Turn the answer into a percent by moving the decimal two places to the right

Answer: 48%

**7**

136 is 16% of what?  
Set up an equation. Remember “what” means “x”, “is” means “=” and “of” means “times”

\[136 = .16 \times x\]
\[136 = .16 \times x\]
\[\div .16 \div .16\]
\[850 = x\]

Answer: 850
If you bought something for $5.50 and there was an 8% sales tax, you would need to find 8% of $5.50 to find out how much tax you were being charged.

\[ 0.08(5.50) = 0.44 \]

If you were going to buy something for $60.99, and there was a 15% discount, you would like to know 15% of $60.99 to find out how much you were going to save.

\[ 0.15(60.99) = 9.15 \]

If you want to buy a $759 computer with 8% sales tax, how much tax will you end up paying?

\[ r = 8\% \]
\[ P = 759 \]
\[ T = ? \]
\[ T = rP \]
\[ T = (0.08)(759) \]
\[ T = 60.72 \]

Answer: You would pay $60.72 in sales tax

How much will Alice save on a pair of shoes that are worth $92 but are on sale for 20% off?

\[ r = 20\% \]
\[ P = 92 \]
\[ D = ? \]
\[ D = rP \]
\[ D = (0.20)(92) \]
\[ D = 18.40 \]

Answer: Alice will save $18.40

The original price of a TV was $75, and it has a 6% sales tax. What is the final price of the TV?

\[ P = 75 \]
\[ r = 0.06 \]
\[ F = ? \]
\[ F = P + T \]
\[ T = rP \]
\[ F = P + rP \]
\[ 75 + 0.06(75) \]
\[ 75 + 4.5 \]

Or in other words:

\[ 75 + 4.5 + 0.06(75) \]

Add the amount of tax to the original price Solve

Answer: The final price is $79.50
Once we have the last concept down, we have the ability to solve tons of problems involving sales tax, mark-ups, and discounts.

**For Example:** An item that sold at $530 has already been marked up 20%. What was the price before the mark-up?

\[ x + 0.2x = 530 \]

original + 20% of original = final price

\[ 1.2x = 530 \]

\[ x = 441.67 \]

Henry purchased a picture frame that was worth $36 for $30.60 after a store discount. What percent discount did he receive?

\[ 36 - x(36) = 30.60 \]

Subtract the amount of discount from the original price and set it equal to the final price

\[ 36 - x(36) = 30.60 \]

Solve the equation for x

\[ -36 \]

\[ -36x = -5.4 \]

\[ \div (-36) \]

\[ x = 0.15 \]

Turn the answer into a percent by moving the decimal point two places to the right.

**Answer:** He received a 15% discount

How many people lived in a town last year if 19,980 people live there this year and it grew 8% from last year?

\[ x + 0.08x = 19,980 \]

Add the amount of growth to last year’s population (x) and set it equal to this year’s population

\[ x + 0.08x = 19,980 \]

\[ 0.08x = 19,980 \]

\[ \div 1.08 \]

\[ x = 18,500 \]

**Answer:** Last year the town’s population was 18,500
This formula is very similar to another one that involves simple interest. If you invested a principal amount of $500 at 9% interest for three years, the amount of interest would be given by the formula:

\[ I = Prt \]

where
- \( I \) is the interest earned
- \( P \) is the principal amount (starting amount)
- \( r \) is the interest rate
- \( t \) is the time that it is invested.

**EXAMPLES**

### Mindy sets up a savings plan that gives her simple interest of 7% per year. If she invests $750, how much interest will she earn in 10 years?

\[ I = Prt \]
\[ r = 7\% \]
\[ P = $750 \]
\[ t = 10 \text{ years} \]

\[ I =? \]
\[ I = (750)(.07)(10) \]
\[ I = 525 \]

Write down what information we have been given

Decide what we are trying to figure out

Plug all of the information that we already know into the equation we are given

Solve for the piece of information that we don’t know yet

Answer: Mindy will earn $525 in interest

### Cooper earned $280 in interest in five years. If he earned 8% interest annually, how much did he invest?

\[ I = Prt \]
\[ I = $280 \]
\[ t = 5 \text{ years} \]
\[ r = 8\% \]

\[ P =? \]
\[ 280 = P(.08)(5) \]
\[ 280 = P(\frac{4}{5}) \]
\[ \div \frac{4}{5} \quad \div \frac{4}{5} \]
\[ 700 = P \]

Write down what information we have been given

Decide what we are trying to figure out

Plug all of the information that we already know into the equation we are given

Solve for the piece of information that we don’t know yet

Answer: Cooper invested $700
1. 45 is 12 more than 3 times a number. What is the number?

2. 25 less than 7 times a number is 108. What is the number?

3. Find the area of the shaded region:

4. If a parallelogram has an area of 258.9 cm$^2$ and a base of 23.2 cm, how tall is it?

5. Find the missing variable for a trapezoid:
   \[ A = 68 \text{ ft}^2 \]
   \[ b = \]
   \[ h = 4\text{ ft} \]
   \[ B = 21\text{ ft} \]

6. \[ 7p + 13 = 33 - 4p \]
7. \[ 5n + 48 = 7n - 2(n - 2) \]
8. \[ 5x - 10 = 7(x - 2) \]

9. \[ 3x - 7 = 12x \]
10. \[ 5x - 7(x + 3) = -2x - 21 \]
11. \[ .06x = 15 - .18x \]

12. \[ .8(7m - 2) = 9.5m + \]
13. \[ .2q - 7 + 2q = 3q - 5 \]
14. \[ 12t = 45 + .4t \]

15. \[ 6(x - 5) - x = 5x - 20 \]
16. \[ 9x - 2(x - 3) = 15x + 7 \]
17. \[ 5x - 13x + x = 7x + 8x \]

18. Two numbers add to 251 and the second is 41 bigger than the first. What are the two numbers?

19. Two numbers add to 336 and the first is 124 bigger than the second. What are the two numbers?

20. I have created a triangular garden such that the largest side is 8m less than twice the smallest and the medium side is 12m larger than the smallest side. If the total perimeter of the garden is 108m, what are the lengths of the three sides?

21. If a rectangle’s length is 5 more than 3 times the width and the perimeter is 58 mm what are the dimensions of the rectangle?
22. 18 is what percent of 58?

23. What is 87% of 54?

24. 34 is 56% of what?

25. What is 13% of 79?

26. 119 is 8% of what?

27. 23 is what percent of 74?

28. Original Price: $92.56
   Tax: 7.3%
   Final Price:

29. Original Price:
   Discount: 40%
   Final Price: $43.90

30. Original Price:
   Tax: 5%
   Final Price: $237.50

31. Original Price: $58.50
   Discount: 30%
   Final Price:

32. If the population of a town grew 21% up to 15,049. What was the population last year?

33. If the price of an object dropped 25% down to $101.25, what was the original price?

**Preparation.**

34. After reading some from Section 3.3, Try to solve this equation.
\[
\frac{4}{7} + \frac{11}{7} = \frac{15}{7} - \frac{2x}{7}
\]

35. Solve.
\[
\frac{x}{3} + \frac{13}{3} = \frac{15}{3} - \frac{2x}{3}
\]
## Answers

1. $x = 11$
2. $x = 19$
3. $136.71 \text{ cm}^2$
4. $11.16 \text{ cm}$
5. $13 \text{ ft} = b$
6. $p = \frac{20}{11}$
7. No solution
8. $x = 2$
9. $x = -\frac{7}{9}$
10. All numbers
11. $x = 62.5$
12. $m = -\frac{2}{3}$
13. $q = -2.5$
14. $t = 3.879$
15. No solution
16. $x = -\frac{1}{8}$
17. $x = 0$
18. 105, 146
19. 106, 230
20. 26m, 38m, 44m
21. $w = 6 \text{ mm}$, $l = 23 \text{ mm}$
22. 31%
23. 46.98
24. 60.7
25. 10.27
26. 1487.5
27. 31%
28. $99.32$
29. $73.17$
30. $226.19$
31. $40.95$
32. 12,437
33. $135$
34. In class
35. In class
As we discussed in Chapter 1, fractions can often seem difficult to people, but they don’t have to be complicated. Even still, sometimes we just want to get rid of them to make things easier to solve. To get rid of fractions in an equation, we will work with some examples that resemble Chapter 1 problems, and others that will use principles you used in the last section with the Balance Rule.

Remember how when we wanted to add fractions, they needed to have the same base, so we changed all of them by multiplying to get a least common denominator (LCD)?

\[
\frac{5}{9} + \frac{1}{3} = \frac{5}{9} + \frac{3}{9} = \frac{8}{9}
\]

When you work across an equals sign, it gets easier because if you find a common denominator, then you can just get rid of it completely. Let us see why:

\[
\frac{5}{6}x + \frac{1}{4} = \frac{11}{3}
\]

Ugly, right? Not really – first find that LCD.

We will need something with a couple twos and a three: 12

To make everything have a base of 12, we use the identity rule and multiply the same on top and bottom:

\[
\frac{2 \cdot 5}{2 \cdot 6}x + \frac{1 \cdot 3}{3} = \frac{11 \cdot 4}{4} \quad \rightarrow \quad \frac{10}{12}x + \frac{3}{12} = \frac{44}{12}
\]
Why does this help? Well, you can either now add the fractions if you want, or you can get rid of the 
12! How? Remember in the Balance Rule that you can do what you want as long as you do the same 
thing to everything on both sides? Well, why don’t we multiply everything by 12, and see what happens:

\[
\frac{12}{12} \cdot \frac{10}{12} x + \frac{3}{12} \cdot \frac{12}{12} = \frac{44}{12} \cdot \frac{12}{12} \\
10x + 3 = 44
\]

Doesn’t that look easier than trying to work with the fractions? And it always works, just follow 
these guidelines…

### Getting Rid of Fractions

1. Determine the LCD.
2. Multiply tops and bottoms to make all denominators the same.
3. Multiply all parts by the denominator to cancel it out.

### EXAMPLES

#### 1

Eliminate the fractions of \(\frac{2}{7} - \frac{3}{4}x = \frac{1}{2}\)

Find the LCD by making a factor tree of the denominators

\[
\frac{2}{7} \times \frac{4}{4} - \frac{3}{4} \times \frac{7}{7} = \frac{1}{2} \times \frac{14}{14} \\
\frac{8}{28} - \frac{21}{28} \times \frac{7}{7} = \frac{14}{28} \times \frac{14}{28}
\]

Multiply the top and bottoms of the fractions by the same numbers to get a denominator of 28

Multiply every part of the equation by the denominator to cancel out all fractions

We are left with an equation without fractions

\[
8 - 21x = 14
\]

#### 2

Eliminate the fractions of \(\frac{2}{3}s - 2 = \frac{1}{2}\)

Find the LCD by making a factor tree of the denominators

\[
\frac{2}{3} \times \frac{2}{2} - \frac{2}{6} = \frac{1}{2} \times \frac{3}{3}
\]

Multiply the top and bottoms of the fractions by the same numbers to get a denominator of 6

Multiply every part of the equation by the denominator to cancel out all fractions

We are left with an equation without fractions

\[
4s - 12 = 3
\]
1. 35 less than 7 times a number is 98. What is the number?

2. The perimeter of a rectangle is 702cm. The length is 71cm longer than the width. What are the dimensions?

Solve.

3. \(7p + 12 = 33 - 4p\)  
4. \(3n + 48 = 7 - 2(n - 2)\)  
5. \(5x - 10 = 5(x - 2)\)  
6. \(3x - 7 = 15x\)  
7. \(5x - 7(x + 3) = -2x + 12\)  
8. \(.09x = 13 - .18x\)  
9. \(.8(3x - 2) = 9.5x + 1\)  
10. \(.2x - 7 + 2x = 3x - 5\)  
11. \(12m = 70 + .4m\)  
12. \(5(x - 5) - x = 4x - 20\)  
13. \(9x - 4(x - 3) = 15x + 7\)  
14. \(8x - 12x + x = 9x + 8x\)

15. I have three colors of paint: blue, green, and yellow. The number of gallons of blue paint is 5 more than twice the number of green. The number of gallons of yellow paint is 3 less than 7 times the number of green. All together I have 82 gallons of paint. How many of each color are there?

16. 85 is what percent of 39?
17. 85 is 54% of what?
18. What is 19% of 2,340?
19. What is 23% of 79?
20. 119 is 18% of what?
21. 43 is what percent of 174?
22. Original Price: $72.56  
   Tax: 7.3%  
   Final Price: 
23. Original Price:  
   Discount: 30%  
   Final Price: $49.70
24. Original Price:  
   Tax: 5%  
   Final Price: $339.50
25. Original Price: $55.50  
   Discount: 40%  
   Final Price:

26. If the population of a town grew 31% up to 17,049. What was the population last year?

27. If the price of an object dropped 35% down to $101.25, what was the original price?
2.5

Example:

\[ \frac{2}{3}(x+4) - 5 = \frac{1}{2}x + \frac{4}{3} \]

\[ \left(\frac{12}{3}\right)(x+4) - 5 = \left(\frac{12}{4}\right)x + \left(\frac{12}{6}\right)\frac{5}{6} \]  Clear fractions by multiplying by 12

\[ 4(x+4) - 30 = 3x + 10 \]

\[ 4x + 16 - 30 = 3x + 10 \]  Distribute through parentheses

\[ x - 14 = 10 \]  Combine, getting x to one side

\[ x = 24 \]  Add 14 to both sides

28.  \[ \frac{7}{3}t - 5 = 19 \]

29.  \[ -\frac{3}{8}(x - 7) = 5 + 3x \]

30.  \[ \frac{2}{3}x - 6 = 3 + \frac{1}{2}x \]

31.  \[ \frac{4}{5}x = 2x - \frac{5}{3} \]

32.  \[ \frac{3}{5}x - \frac{2}{5}(x - 3) = \frac{1}{5}x + 3 \]

33.  \[ \frac{3x+2}{7} = \frac{4x-1}{5} \]

34.  \[ .9(-4x - 5) = 2.5x + 6 \]

35.  \[ .0005x + .0045 = .004x \]

36.  \[ \frac{x+7}{4} = 8 - \frac{5}{6}x \]

Preparation.

37. Solve.

\[ 3x - 7 = 17 \] \[ 3x - 7 < 17 \] \[ 3x - 7 > 17 \]
Answers
1. 19
2. $w = 140\text{cm}, l = 211\text{cm}$
3. $p = \frac{21}{11}$
4. $n = -\frac{17}{3}$ or -7.4
5. All numbers
6. $x = -\frac{7}{12}$
7. No solution
8. $x = 48.15$
9. $x = -0.366$
10. $x = -2.5$
11. $m = 6.03$
12. No solution
13. $x = \frac{1}{7}$
14. $x = 0$
15. 21 gal – blue, 8 gal – green, 53 gal – yellow
16. 218%
17. 157.4
18. 444.6
19. 18.17
20. 661.1
21. 24.7%
22. $77.86$
23. $71.00$
24. $323.33$
25. $33.30$
26. 13,015
27. $155.77$
28. $t = \frac{22}{7}$
29. $x = -\frac{19}{27}$
30. $x = 54$
31. $x = \frac{25}{18}$
32. No solution
33. $x = \frac{17}{13}$
34. $x = -\frac{105}{61}$
35. $x = \frac{9}{7}$
36. $x = \frac{75}{13}$
37. In class.
With this section we begin a new concept that involves things not being just equal, but also greater or less than each other.

**Symbols of Inequalities**

When using inequalities, there are 4 different symbols with which you need to be familiar:

- `<` Less than / Greater than
- `≤` Less / Greater than or Equal to

Either of these symbols can be flipped to point in either direction, depending on what you want to say. For example, we know that:

\[
5 < 10
\]

So why does the symbol point at the 5? Because the arrow means small to big:

smaller ⟹ BIGGER

If what is represented by the inequality is true, then it is called a **Solution**. Here are some examples:

\[
\begin{align*}
4 & > -2 \\
-59 & > -360 \\
159 & < 2,198 \\
2.98 & > 1.6 \\
\frac{5}{9} & < \frac{7}{8} \\
\frac{6}{5} & > \frac{2}{3}
\end{align*}
\]

Now that you understand the symbol, we can use them to represent the relationship a variable may have with a number. For example, if I were to say “I am taller than 5 feet,” does that tell you how tall I am? No, but it does give you a range, because you know I’m not 4 feet tall. You can write that like this:

My height > 5 feet or \( x > 5 \) where “x” is my height.
1. **Represent this as an inequality: I have less than $50 in my wallet.**

- Money in my wallet: $50
- Put the two quantities side by side
- Assign a variable to the one that we don’t know the value for
- Decide which direction the inequality sign will point.
- Remember it points to the smaller number!

2. **Represent this as an inequality: There are more than 30 people in the class.**

- People in my class: 30
- Put the two quantities side by side
- Assign a variable to the one we don’t know the value for
- Decide which direction the inequality sign will point.
- Remember it points to the smaller number!

DEFINITIONS & BASICS

The other symbol with the line underneath is used when representing things that are equal to or less than / greater than a number, like when we say “at least” or “at most.”

Example: I need at least $20 to buy the shirt. $x \geq 20$

The line underneath the symbol means that “$x$” can actually be $20$, but it can also be more than 20. The statement suggests that if I have $20$, that would be enough. If I have more, I can still buy the shirt. In other words:

\[ 20 \leq 20 \quad \text{and} \quad 20 \leq 25 \quad \text{are both true, but} \quad 20 \leq 18 \quad \text{is false.} \]

3. **Represent this as an inequality: There is at most $500 left in my bank account.**

- Money in my bank account: $500
- Put the two quantities side by side
- Assign a variable to the one we don’t know the value for
- Decide which direction the inequality sign will point.
- Remember it points to the smaller number!

4. **Represent this as an inequality: I had at least seven compliments on my presentation.**

- Compliments I received: 7
- Put the two quantities side by side
- Assign a variable to the one we don’t know the value for
- Decide which direction the inequality sign will point.
- Remember it points to the smaller number!
A graph of an inequality uses a number line that represents all of the solutions possible. A basic example is:

\[ x > 2 \]

so \( x \) can be any number greater than 2:

The open circle means that 2 is NOT included (since “\( x \)” cannot be equal to 2).

The arrow means that everything above is included in the solution.

Here is another example:

\[ x \leq 1 \]

Notice that the circle is filled in because “\( x \)” can be equal to 1.

You can also represent two number solutions on a single graph. For example, if \( x \) is greater than 2, but less than or equal to -2, you can represent it like this:

\[ x \leq -2 \text{ or } 2 < x \]

The word “or” means that either solution works.

The word “and” means that BOTH solutions have to be true.

NOTE: Since “and” was used, the last solution can be written like this:

\[ -3 \leq x \text{ and } x \leq 4 \]

These are called compound inequalities.

EXAMPLES

Graph the following on a number line: \( x < -4 \text{ or } x \geq 1 \)

Graph the inequality piece by piece
A couple will be going on a trip. They invited 5 other people to go with them. Not every one of their friends may be able to come. Represent the amount of people that could go on the trip with inequalities, and then graph it with a number line.

\[ x \geq 2 \text{ and } x \leq 7 \]

We know \( x \) is greater than or equal to 2 because at least the couple will be going.

We know \( x \) will be less than or equal to 7 because at most all 5 guests and the couple will go on the trip.

Graph the inequality on a number line.

C SOLVING INEQUALITIES: ARITHMETIC AND THE RULE OF NEGATIVES

DEFINITIONS & BASICS

As far as solving inequalities, they are exactly the same as solving equations, except for one thing. Let’s try to solve this:

\[
\frac{-3x \leq 6}{-3} \quad \text{so: } \quad x \leq -2
\]

This means that numbers such as -4, and -18 should be answers, but when we plug them back in:

\[
-3(-4) \leq 6 \quad \text{is not a true statement}
\]
\[
-3(-18) \leq 6 \quad \text{is not a true statement either.}
\]

But, look and see that numbers such as -1 and 8 work:

\[
-3(-1) \leq 6 \quad \text{is true}
\]
\[
-3(8) \leq 6 \quad \text{is also true.}
\]

It is like we got exactly the wrong half of the number line. Now see what happens when we start dividing or multiplying by negative numbers:

\[
8 > 5 \quad \text{is a true statement; so if we multiply both sides by -2:}
\]
\[
-16 \quad ? \quad -10 \quad \text{which direction should the sign go?}
\]

Obviously \(-16 < -10\), so we start to see that when we make things negative, the direction of the inequality switches. Here is one with division:

\[
-4 < 22 \quad \text{We divide both sides by -2:}
\]
\[
2 \quad ? \quad -11 \quad \text{which way should the inequality go?}
\]

Because \(2 > -11\) we see that:

**Multiplying or dividing by a negative number switches the sign’s direction.**
Now to get back to our original problem that started this discussion:

\[-3x \leq 6\]

\[\frac{-3x}{-3} \leq \frac{6}{-3}\]

\[x \geq -2\]

The division by a negative makes the direction change.

So the answer is:

\[x \geq -2\]

and the graph is

\[\text{Graph the answer}\]

EXAMPLES

7 Solve this inequality for \(x\) and graph the answer: \(4x + 2 - 5x \geq 10\)

Combine all like terms

Get all of the \(x\)’s on one side and all of the numbers on the other

Divide to get \(x\) all alone

Switch the direction of the inequality sign if we divide by a negative number

Graph the answer

8 Solve this inequality, and then write it in interval notation and graph:

\[2(3 + 4y) - 9 \geq 45\]

Distribute into the parenthesis

Combine all like terms

Get all of the \(y\)’s on one side and all of the numbers on the other

Divide to get \(y\) all alone

Don’t switch the sign because we did not divide by a negative number

Graph the answer

Section 2.6
Solve for the specified variable.

2.2
1. \( \frac{2x - at^2}{2t} = V \) for \( s \)
2. \( r = \frac{l}{pt} \) for \( p \)
3. \( d = \frac{LR_2}{R_2 + R_1} \) for \( R_1 \)
4. \( \frac{9s - 5g}{11} = c \) for \( s \)

5. 84 is 6 more than 3 times a number. What is the number?

6. A stick that is 438 cm long is cut into two pieces. The first is 74 bigger than the second. What are the lengths of the two pieces?

7. Find the area of the shaded region:

8. If a rectangle’s length is 7 more than 4 times the width and the perimeter is 194 mm, what are the dimensions of the rectangle?

9. Find the missing variable for a rectangle:
   \( P = 48.3 \) ft
   \( w = 7.2 \) ft
   \( l = \)

10. Find the missing variable for a sphere:
    \( SA = 800 \) in\(^2\)
    \( r = \)

Solve.

2.3
11. \( 7p + 12 = 13 - 7p \)
12. \( 4n + 68 = 7 - 2(n - 2) \)
13. \( 7x - 10 = 5(x - 2) \)
14. \( 9x - 4 = 15x \)
15. \( 8x - 7(x + 3) = x - 21 \)
16. \( .18x = 13 - .20x \)

2.4
17. 119 is 18% of what?
18. 27 is what percent of 74?
19. Original Price: \$192.56
   Tax: 7.3%
   Final Price:
20. Original Price:
    Discount: 35%
    Final Price: \$43.90
21. If the price of a meal after a 20% tip was $28.80? What was the price of the meal before the tip was added?

22. If the price of an object dropped 15% down to $59.50, what was the original price?

Solve.

23. \( \frac{7}{3} t - 2 = 19 + 5t \)
24. \( -\frac{3}{4} (x - 4) = 5 + 2x \)
25. \( \frac{1}{6} x - 4 = 3 + \frac{3}{10} x \)
26. \( \frac{5}{2} (-4x - 2) = \frac{3}{4} x + 6 \)
27. \( \frac{x-5}{3} = \frac{5x+8}{6} \)
28. \( \frac{x+7}{14} = 6 - \frac{3}{7} x \)

Solve and Graph

29. \( 3t + 5 > 12 \)
30. \( 4m + 2 \leq -18 \)
31. \( 3(x + 4) - 6x \geq 5(x - 2) \)
32. \( -7p + 3 < 24 \)
33. \( \frac{3}{5} (n + 4) - 2 \geq \frac{3}{2} n \)
34. \( 3m < -21 \)
35. \( \frac{3}{4} (x - 5) \geq \frac{7}{2} x + 1 \)
36. \( 3a + 5a \leq 7a - 8a \)
37. \( 5y - 7 \geq \frac{2}{3} y + 4 \)

38. At a family reunion, Logan reserves a table at a dinner and a show event. There is a $50 reservation fee for the show, plus a fee of $15 per person for the dinner. If he has a budget of $450, how many people can come to the dinner?

39. On his first two tests, Josh received scores of 85 and 89. If he wants at least a 90 for the average of his first three tests, what possible scores could he get on his third test?
Answers:

1. \( s = \frac{2Vt + at^2}{2} \)

2. \( p = \frac{I}{rt} \)

3. \( R_1 = \frac{LR_2 - dR_2}{d} \)

4. \( s = \frac{11c + 5g}{9} \)

5. 26

6. 182 cm, 256 cm

7. 62.38 in²

8. 18 mm X 79 mm

9. \( l = 16.95 \text{ ft} \)

10. 7.98 in

11. \( p = \frac{1}{14} \)

12. \( n = -9.5 \)

13. \( x = 0 \)

14. \( x = -\frac{2}{3} \)

15. All numbers

16. \( x = 34.21 \)

17. 661.1

18. 36.5%

19. $206.62

20. $67.54

21. $24

22. $70

23. \( t = -\frac{63}{8} \) or -7.875

24. \( x = -\frac{8}{11} \) or -0.73

25. \( x = -52.5 \)

26. \( x = -\frac{44}{43} \) or -1.02

27. \( x = -6 \)

28. \( x = 11 \)

29. \( t > \frac{7}{3} \)

30. \( m \leq -5 \)

31. \( x \leq \frac{11}{4} \)

32. \( p > -3 \)

33. \( n \leq \frac{4}{9} \)

34. \( m < -7 \)

35. \( x \leq -\frac{19}{11} \)

36. \( a \leq 0 \)

37. \( y \geq \frac{33}{13} \)

38. Number \( \leq 26 \)

39. \( t \geq 96 \)
We will now expand our word problems to dealing with inequalities, in very similar ways that we did problems in Section 5.

A  INEQUALITY WORD PROBLEMS: TRANSLATION TERMINOLOGY

To make translation simple, we will present some of the more common phrases used in inequality problems; these are phrases for which you should pay attention:

<table>
<thead>
<tr>
<th>COMMON PHRASES</th>
<th>EXAMPLE</th>
<th>TRANSLATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least</td>
<td>I need at least $10.</td>
<td>$x \geq 10$</td>
</tr>
<tr>
<td>at most</td>
<td>You can have at most 15 in the group.</td>
<td>$P \leq 15$</td>
</tr>
<tr>
<td>cannot exceed / maximum</td>
<td>The weight cannot exceed 2000lbs.</td>
<td>$W \leq 2000$</td>
</tr>
<tr>
<td>must exceed / minimum</td>
<td>The minimum speed is 35 mph.</td>
<td>$S \geq 35$</td>
</tr>
<tr>
<td>less than</td>
<td>He has less than a 95%</td>
<td>$x &lt; 95%$</td>
</tr>
<tr>
<td>more than</td>
<td>We have more than $1000 in our account.</td>
<td>$M &gt; 1000$</td>
</tr>
<tr>
<td>between</td>
<td>His age is somewhere between 25 and 30 years old.</td>
<td>$25 &lt; A &lt; 30$</td>
</tr>
<tr>
<td>no more than</td>
<td>There can be no more than 10 people on the boat.</td>
<td>$P \leq 10$</td>
</tr>
<tr>
<td>no less than</td>
<td>You should walk no less than 5 miles a day.</td>
<td>$d \geq 5$</td>
</tr>
</tbody>
</table>

These are not the only ways that you can express these ideas, but they are the most common ones. Be sure to realize which ones indicate a “less than or equal to” as opposed to just a “less than.”

EXAMPLES

1. In order for my model rocket to work it needs to put out a power of at least 10 horsepower.
   …power of “at least” 10 horsepower.  
   Power $\geq 10$ horsepower
   
   Determine the key phrase that will help us to figure out which inequality sign to use
   Translate it into an inequality sign
   
   **Answer:** $Power \geq 10$ horsepower

2. The required thickness of penny is between 1.8 and 2mm.
   …thickness of penny is “between” 1.8 and 2mm.
   $1.8mm \leq Thickness \leq 2mm$
   
   Determine the key phrase that will help us to figure out which inequality sign to use
   Translate it into an inequality sign
   
   **Answer:** $1.8mm \leq Thickness \leq 2mm$
Glen Road has a speed limit of 45mph.

Determine the key phrase that will help us to figure out which inequality sign to use

Translate it into an inequality sign

Answer: \( \text{speed} \leq 45\text{mph} \)

There is a tendency, when solving inequalities, to forget what we are talking about. You can’t just regurgitate an answer once you solve for it – you need to ask the question “does this make sense?” This is a good proof for just about any problem you are solving, but with inequalities, this is essential. Let’s use an example to explain this principle:

A group of students have planned a field trip to a science museum. It costs $50 to rent a bus for the day, and it will cost $12 per student to enter the museum and buy lunch. The group has a budget of $450, which they cannot exceed. How many students can go on the trip?

We can represent the problem with the following inequality:

\[
450 \geq 12s + 50
\]

\[
\therefore 33 \frac{1}{3} \geq s
\]

When we solve, we show that \(33 \frac{1}{3}\) students can come – but can you really have a third of a student? No! Therefore our answer would be “at most, 33 students can go.”

A box that weighs 5 lbs can hold up to 25 books that each weighs 2.5 lbs. Due to recent back surgery I can only carry at most 48 lbs. If I want to move the box, how many books can the box have in it?

The weight of the box (5lbs) plus 2.5 lbs for every book (x) can be at most 48 lbs.

\[
5 + 2.5x \leq 48
\]

\[
\therefore 2.5x \leq 43
\]

\[
\frac{2.5x}{2.5} \leq \frac{43}{2.5}
\]

\[
x \leq 17.2
\]

Can we have 17.2 books? No, so we need to change it to 17

\[
x \leq 17
\]

Answer: Number of books \( \leq 17 \)
In order for David to reach his saving goal he needs to earn $109,200 in commission this year. He earns 15% commission from all of his sales. If he earned $89,700 in commission last year, by how much does he need to increase his sales this year in order to reach his goal?

David’s sales amount last year ($89,700) needs to increase by 15% of x to be at least $109,200

\[ 89,700 + 0.15x \geq 109,200 \]

-89,700 - 89,700

\[ 0.15x \geq 19,500 \]

\[ x \geq 130,000 \]

Can we have 130,000 dollars? Yes, our answer works

**Answer**: increase in sales \( \geq \$130,000 \)

---

Cindy is trying to make a batch of her grandmother’s cookies. The problem is that her grandma never wrote down the recipe. She did tell Cindy that the cookies need between 3 and 4 ½ cups of flour. If Cindy has already added \( \frac{1}{4} \) cup of flour, how much more does she need to add?

The total amount of flour in the cookies (\( \frac{1}{4} + x \)) needs to be between 3 and 4 ½ cups

\[ 3 \leq \frac{1}{4} + x \leq 4 \frac{1}{2} \]

\[ 3 \leq \frac{1}{4} + x \leq 4 \frac{1}{2} \]

\[ \frac{1}{4} \leq x \leq 4 \frac{1}{2} \]

\[ 2 \frac{3}{4} \leq x \leq 4 \frac{1}{4} \]

Can we have between 2 ¾ and 4 ¼ cups of flour? Yes, our answer works

**Answer**: \( 2 \frac{3}{4} \leq \) more flour \( \leq 4 \frac{1}{4} \)
Translate the following statements into an equation and solve.

1. 56 is what percent of 448?
2. What is 15% of 0.0012?
3. 421 is 105.25% of what?

4. While studying the weather patterns in Omaha, Jackson recorded that between the months of March and May the average temperature highs rose by 25%. If the average temperature in May is 78°F, what was the average temperature in March?

5. While running her latest marathon, Erika lost 2% of her body weight in sweat. After drinking water after the race, she regained 90% of the weight that she lost. If she originally weighed 120 pounds, how much did she weigh after drinking?

Solve the following inequalities and represent the answer using interval notation.

6. $14h + (-7.2) < -220$
7. $8k - \frac{3}{4} \geq 10k$
8. $0.5(8r + 22) \leq 2r + 11$
9. $\frac{75}{3} > \frac{(15b - 30)}{3}$
10. $12.7p + 4.5 + 1.3p \geq -31.9$

Write the following statements as an inequality.

11. It is at least 100° outside.
12. The show will begin in less than 5 minutes.
13. 60 inches is the minimum height to enter.
14. The road is between 5 and 8 miles away.
15. The speed limit is 45 miles per hour.
16. I have more than 3 years until graduation.

Solve the following word problems by writing them in an inequality.

17. On his first two tests, Josh received scores of 88 and 92. If he wants at least an average of 93, what does his score on the third test have to be?

18. An elevator can hold up to 3,300 pounds. If each person on the elevator weighs an average of 165 pounds, how many people can ride at one time?

19. At a family reunion, Logan reserved a table at a dinner and a show event. There was a $50 reservation fee for the show, plus a fee of $15 per person for the dinner. If he had a budget of $450, how many people can come to the dinner?

20. In order to qualify for financial aid, Sheyla needs to take at least 30 credits combined between two semesters. If she took 16 credits last semester, how many credits does she need to take to qualify for aid?

21. Patty wants to know how long she can talk to her grandma on a long distance phone call with the $2.20 she has. If it costs $0.50 to place a call and $0.10 per minute, how long can she talk?

22. The width of a rectangle is fixed at 6 meters. For what lengths will the area be more than $96m^2$?
Answers:
1. 12.5%
2. 0.00018
3. 400
4. 62.4° F
5. 119.76 pounds
6. \( h < -15.2 \)
7. \( k \leq -0.375 \)
8. \( r \leq 0 \)
9. \( b < 7 \)
10. \( p \geq -2.6 \)
11. Temperature \( \geq 100° \)
12. Show \( < 5 \) minutes
13. height \( \geq 60 \) inches
14. 5 miles \( \leq \) road \( \leq 8 \) miles
15. speed \( \leq 45 \) mph
16. years \( > 3 \)
17. grade \( \geq 99 \)
18. people \( \leq 20 \)
19. people \( \leq 26 \)
20. credits \( \geq 14 \)
21. time \( \leq 17 \) minutes
22. length \( > 16 \) meters
2.1 STEPS 2 & 3--The Addition and Multiplication Principles

### Checking a Solution
1. Plug the solution into the equation
2. Simplify the equation using the order of operations
3. If the result is a true statement, the answer is a solution
   If the result is a false statement, the answer is not a solution

### Balance Rule of Solving:
Whatever I do to one side of the equal sign, I must do the exact same to the other side to maintain equality.

### 3-Step Process to Solving
**GOAL:** Get $x$ alone ($x$ will represent any variable)
1. **SIMPLIFY**
   A) Get rid of fractions (multiply all by LCD)
   B) Distribute across (    )
   C) Combine “like terms” and get all $x$’s on same side
2. **ADDITION PRINCIPLE**
3. **MULTIPLICATION PRINCIPLE**

### Summary of Addition & Multiplication Principles:
1. Identify: Who’s sharing sides with $x$?
2. Identify: What are they doing to $x$? (add, subt, mult, or divide?)
3. Do the opposite (operation) to both sides of the equation.

---

### 2.2 Applications and Formulas

**D- Data.** Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

**V- Variable.** In all of these story problems, there is something that you don’t know, that you would like to. Pick any letter of the alphabet to represent this.

**P- Plan.** Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. This section of the book is divided up so as to explain most of the different kinds of patterns.

**E- Equation.** Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

Review the formulas from Chapter 1, including shapes, simple interest, distance, etc.
## 2.3 STEP 1—Simplify the Equation: Parentheses, Like Terms

### Simplifying Solve Problems

1. Combine like terms on the left of the = sign
2. Combine like terms on the right of the = sign
3. Get all your x’s (variables) to one side.

<table>
<thead>
<tr>
<th>Solution is <strong>all real numbers</strong> if you get something like:</th>
<th>The Distributive Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 = 0</td>
<td>The term outside the parentheses needs to touch each term inside the parentheses.</td>
</tr>
<tr>
<td>5 = 5</td>
<td>3(2x + 4) = 6x + 12</td>
</tr>
<tr>
<td>-3 = -3</td>
<td>There is <strong>no solution</strong> if you get something like:</td>
</tr>
<tr>
<td></td>
<td>0 = 1</td>
</tr>
<tr>
<td></td>
<td>5 = 7</td>
</tr>
<tr>
<td></td>
<td>-3 = 2</td>
</tr>
</tbody>
</table>

## 2.4 Applications: Percents

### Substitution

Write a relationship between two of the variables, and substitute one of the variable into the formula.

\[
\begin{align*}
    w &= 4 \\
    l &= 2w - 3
\end{align*}
\]

\[
P = 2w + 2l
\]

\[
P = 2(4) + 2(2w - 3)
\]

### Simple Percent

- **WHAT** means x
- **IS** means =
- **OF** means times (\(\times\))

118 is what percent of 230?

\[
118 = x \times 230
\]

Don’t forget to change percents into decimals!

## 2.5 STEP1—Simplify the Equation: Fractions, Parentheses, Like Terms

### Getting Rid of Fractions

1. Determine the LCD.
2. Multiply tops and bottoms to make all denominators the same.
3. Multiply all parts by the denominator to cancel it out!
2.6 Inequalities

Graphing Inequalities
- A solid point indicates that that number is included in the set: \( \leq \) or \( \geq \)
- A hollow point indicates that the set includes everything on the ray except that number: \(<\) or \(>\)

Remember:
Multiplying or dividing by a negative number switches the sign’s direction.

2.7 Word Problems: Inequalities

<table>
<thead>
<tr>
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<tr>
<td>at least</td>
<td>( x \geq 10 )</td>
</tr>
<tr>
<td>at most</td>
<td>( P \leq 15 )</td>
</tr>
<tr>
<td>cannot exceed / maximum</td>
<td>( W \leq 2000 )</td>
</tr>
<tr>
<td>must exceed / minimum</td>
<td>( S \geq 35 )</td>
</tr>
<tr>
<td>less than</td>
<td>( x &lt; 95% )</td>
</tr>
<tr>
<td>more than</td>
<td>( M &gt; $1000 )</td>
</tr>
<tr>
<td>between</td>
<td>( 25 &lt; A &lt; 30 )</td>
</tr>
<tr>
<td>no more than</td>
<td>( P \leq 10 )</td>
</tr>
<tr>
<td>no less than</td>
<td>( d \geq 5 )</td>
</tr>
</tbody>
</table>
1. Create a visual chart of all of the methods, formulas, and examples from studying how to solve these linear equations.

Solve.

2. \(-\frac{3}{5} m = 24\)  
3. \(9 \left( \frac{8 - 6x}{4} + 6 \right) + 5 = -31\)
4. \(\frac{8x - 5}{3} = 33\)

Solve for the specified variable.

5. \(d = \frac{pM(f - t)}{R}\) for \(t\)  
6. \(d = \frac{pM(f - t)}{R}\) for \(R\)

7. 13.2 less than 7 times a number is 18.8. What is the number?

8. Two numbers add to 336 and the first is 24 bigger than the second. What are the two numbers?

9. Find the area of the shaded region:

![Shaded Region Diagram]

10. I have created a triangular garden such that the largest side is 6m less than twice the smallest and the medium side is 15m larger than the smallest side. If the total perimeter of the garden is 105m, what are the lengths of the three sides?

11. If a parallelogram has an area of 158.9 cm² and a base of 23.2 cm, how tall is it?

12. Find the missing variable for a trapezoid:

\[A = 76 \text{ ft}^2\]
\[b = \_\]
\[h = 4\text{ ft}\]
\[B = 23\text{ ft}\]

Solve.

13. \(7p + 12 = 12 + 7p\)
14. \(9n + 48 = 7n - 2(n - 2)\)
15. \(7x + 18 = 9(x - 3)\)

16. 45 is what percent of 39?

17. 25 is 44% of what?

18. What is 59% of 2,340?

19. What is 83% of 79?

20. Original Price:  
   Tax: 5%  
   Final Price: $359.50

21. Original Price: $55.50  
   Discount: 20%  
   Final Price:  

Chapter 2 Review 1
22. If the population of a town grew 11% up to 17,046. What was the population last year?

23. If the price of an object dropped 15% down to $62.90, what was the original price?

Solve.

24. \( \frac{7}{3} t - 8 = 4 + 7t \)  
25. \( -\frac{3}{7} (m - 12) = 3m + 6 \)  
26. \( \frac{5}{6} x - 8 = 7 + \frac{7}{8} x \)

27. \( .13(-2x + 2) = .05x + 7 \)  
28. \( \frac{x - 7}{4} = \frac{5x + 3}{10} \)

Solve and graph.

29. \( 3t + 5 > 15 \)  
30. \( 4m + 30 \leq -18 \)  
31. \( 3(x + 2) - 6x \geq 5(x - 2) \)

32. \( -7p + 3 < -10 \)  
33. \( \frac{3}{5} (n + 6) - 2 \geq \frac{3}{2} n \)  
34. \( 7m < -21 \)

35. \( \frac{3}{4} (x - 5) \geq \frac{7}{2} x + 15 \)  
36. \( 3a + 5a > 7a - 8a \)  
37. \( 5y - 15 \geq \frac{2}{3} y + 4 \)

38. A copy job to run a pamphlet costs $7 for a setup fee and then $2.21 for each copy. How many copies can be run if the budget is $175?

39. An envelope has to have less than 18 in\(^2\) total area. What can the length be if the width is \( 3\frac{1}{4} \) in?
Answers:
1. It better be good.
2. \( m = -108 \)
3. \( x = 8 \)
4. \( x = 13 \)
5. \( t = \frac{pMf - Rd}{pM} \) or \( t = f + \frac{Rd}{pM} \)
6. \( R = \frac{pM(f - t)}{d} \)
7. \( \frac{32}{7} \) or 4.57
8. 156, 180
9. 282.16 cm²
10. 24 m, 39 m, 42 m
11. 6.85 cm
12. \( b = 15 \) ft.
13. All numbers
14. \( n = -11 \)
15. \( x = \frac{45}{7} \) or 22.5
16. 115.4%
17. 56.82
18. 1380.6
19. 65.57
20. $342.38
21. $44.40
22. 15,357
23. $74
24. \( t = -\frac{18}{7} \)
25. \( m = -\frac{1}{4} \)
26. \( x = -360 \)
27. \( x = -21.74 \)
28. \( x = -\frac{41}{5} \) or -8.2
29. \( t > \frac{10}{3} \)
30. \( m \leq -12 \)
31. \( x \leq 2 \)
32. \( p > \frac{13}{7} \)
33. \( n \leq \frac{16}{9} \)
34. \( m < -3 \)
35. \( x \leq -\frac{75}{11} \)
36. \( a > 0 \)
37. \( y \geq \frac{57}{13} \)
38. pamphlets ≤ 76
39. length ≤ 5.5 in
Solve.

1. \( \frac{4}{7} j = 8 \)

2. \( 9 \left( \frac{3 + 9x}{5} - 8 \right) + 14 = -4 \)

3. \( \frac{12x - 9}{3} = 33 \)

**2.1** Solve for the specified variable.

4. \( m = \frac{BY(U + I) + d}{a} \) for I

5. \( m = \frac{BY(U + I) + d}{a} \) for d

6. 1.4 more than 14 times a number is 112. What is the number?

7. The larger number minus the smaller number is 66 and the larger number is 180 less than twice the smaller number. What are the two numbers?

8. Find the area of the shaded region:

9. I have created a triangular dog kennel such that the largest side is 1ft more than four times the smallest and the medium side is 1ft more than the three times the smallest side. If the total perimeter of the kennel is 34ft, what are the lengths of the three sides?

10. If a parallelogram has an area of 94.38 cm\(^2\) and a height of 14.3 cm, how long is the base?

11. Find the missing variable for a trapezoid:

\[
A = 28.76 \text{ ft}^2 \\
b = 3.6 \text{ ft} \\
h = \\
B = 4.5 \text{ft}
\]

12. 3U + 4(2U – 1) = 10U + 10

13. 3S – 2 = 1 – 4(S + 2) + 7S

14. A + 1 = A + (3 – 2)

15. 81.9 is what percent of 39?

16. 31 is 62% of what?

17. What is 28% of 3264?

18. What is 12% of 624?

19. Original Price:
   Tax: 6%
   Final Price: $104.94

    Discount: 30%
    Final Price:
21. If the population of a wild dingoes grew 44% up to 56. What was the population last year?

22. If the price of an object dropped 20% down to $62.90, what was the original price?

Solve.

23. \( \frac{8}{3} t + 2 = 4 + t \)

24. \( \frac{1}{4} (m + 3) = 2m - \frac{5}{12} \)

25. \( \frac{9}{16} x + 2 = 12 - \frac{31}{16} x \)

26. \( .11(6x - 97) = .26x + 12 \)

27. \( 2x - 17 = \frac{3x+13}{7} \)

Solve and graph.

28. \( 2t + 3 > 11 \)

29. \( 5m - 12 \leq 7 \)

30. \( 4(x - 2) + x \geq 5(x - 8) \)

31. \( 6p - 2 < -10 \)

32. \( \frac{4}{7} (n - 2) + 1 \geq \frac{1}{2} n \)

33. \( m + 3 > 0 \)

34. \( \frac{2}{3} (2x + 3) \geq x + 18 \)

35. \( 2a - 5a \leq -8a \)

36. \( \frac{1}{3} y - 15 \geq \frac{7}{9} y + 4 \)

37. Janet has budgeted $135 to retile her bathroom. The tile she wants to purchase online is $2.05 per tile along with a $21.50 shipping and handling fee. How many tiles can be purchased?

38. A box has to have more than 12 in\(^3\) total volume. What can the height be if the length of the base is 4 in and the width of the base is 2 in?
Answers:
1. $j = 14$
2. $x = 3$
3. $x = 9$
4. $I = \frac{ma - d}{BY} - U$
5. $d = ma - BY(U + I)$
6. 7.9 or $\frac{79}{10}$
7. 246,312
8. 302.16cm²
9. 4ft, 13ft, 17ft
10. 6.6 cm
11. $h = 7.1$ft
12. $U=14$
13. No Solution
14. $A=All$ numbers
15. 210%
16. 50
17. 913.92
18. 74.88
19. $99.00$
20. $44.09$
21. 39
22. $78.63$
23. $t = \frac{6}{3}$
24. $m = \frac{2}{3}$
25. $x = 4$
26. $x = 56.68$
27. $x = 12$
28. $t > 4$
29. $m \leq \frac{19}{5}$
30. $x = all$ numbers
31. $p < \frac{-4}{3}$
32. $n \geq 2$
33. $m > -3$
34. $x \geq 48$
35. $a \leq 0$
36. $y \leq \frac{-171}{4}$
37. tiles $\leq 55$
38. height $> 1.5$ in
Chapter 3: LINES

Overview

3.1 Graphing
3.2 Slope
3.3 Graphing with Slope: Standard and Slope-Intercept Forms
3.4 Writing Equations
3.1 Graphing

**OBJECTIVES**
- Introduce lines
- Learn the “Pick & Stick” method of finding points
- Graph solutions to equations
- Graph using x- and y-intercepts
- Graph horizontal and vertical lines

**A LINES (Overview)**

**CHAPTER 3 TOPICS**

<table>
<thead>
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<td>Parallel = Same</td>
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</tr>
</tbody>
</table>

**EXAMPLES**

- **Equation:** $-2x + y = 1$
  - **Table:**
    | x   | y  |
    |-----|----|
    | -1  | -1 |
    | 0   | 1  |
    | 1   | 3  |
    | 2   | 5  |

- **Equation:** $3x + 2y = 4$
  - **Table:**
    | x   | y  |
    |-----|----|
    | -2  | 5  |
    | 0   | 2  |
    | 2   | -1 |
    | 4   | -4 |
When we solved equations that looked like $3x-2=13$, we got a solution like $x=5$. There is no other number for ‘x’ that will work for this equation. We call this a solution to the equation. So how do we deal with this next type of equation?

$$3x + 2y = 5$$

In this type of equation there is an ‘x’ and a ‘y’. The solution to this equation will not be a single number as it was in the earlier cases, but pairs of numbers. The answers will look like (3,-2), which means that we will stick in 3 for x and -2 for y. If you stick those in, the equation becomes:

$$3(3) + 2(-2) = 5$$
$$9 - 4 = 5$$

It works! Let’s see if there are other solutions:

- (1,1) $3(1) + 2(1) = 5$ solution
- (3,2) $3(3) + 2(2) = 5$ nope
- (-1,4) $3(-1) + 2(4) = 5$ solution
- (5,-2) $3(5) + 2(-2) = 5$ nope
- (5,-5) $3(5) + 2(-5) = 5$ solution

As you can see, some pairs work as solutions and some don’t. In any case, you should be able to realize that there are a whole lot of solutions; really there are an infinite number of them!

There’s an easy way to find solutions to an equation: if we pick a number for ‘x’ and stick it in, then we will have an equation to solve for ‘y’.

**“Pick & Stick”**

1. Pick any number for ‘x’ (or ‘y’)
2. Stick it in
3. Solve for ‘y’ (or ‘x’)
4. Graph point
5. Repeat as desired

*We will discuss graphing the points in the next subsection; for now, we will demonstrate the “Pick & Stick” method a few times:
Find 4 solutions to the following equation: \(16 + 3x = 2y\)

\[\begin{align*}
x = 2 & \rightarrow 16 + 3(2) = 2y \\
& \quad 22 = 2y \\
& \quad 11 = y \\
& \quad \quad (2, 11) \\
\text{Pick any number for x or y} \\
\text{Stick it into the equation and solve for the other variable.}
\end{align*}\]

\[\begin{align*}
y = 1 & \rightarrow 16 + 3x = 2(1) \\
& \quad 3x = -14 \\
& \quad x = -\frac{14}{3} \\
& \quad \quad \left(-\frac{14}{3}, 1\right) \\
\text{Put both the number we chose for x or y and the number we solved for into an ordered pair (x,y)}
\end{align*}\]

\[\begin{align*}
x = -1 & \rightarrow 16 + 3(-1) = 2y \\
& \quad 13 = 2y \\
& \quad 6.5 = y \\
& \quad \quad (-1, 6.5) \\
x = 0 & \rightarrow 16 + 3(0) = 2y \\
& \quad 16 = 2y \\
& \quad y = 8 \\
& \quad \quad (0, 8)
\end{align*}\]

Find 3 solutions to the following equation: \(\frac{5}{3}x + y = 4\)

\[\begin{align*}
x = 3 & \rightarrow \frac{5}{3}(3) + y = 4 \\
& \quad 5 + y = 4 \\
& \quad y = -1 \\
& \quad \quad (3, -1) \\
\text{Pick any number for x or y} \\
\text{Stick it into the equation and solve for the other variable.}
\end{align*}\]

\[\begin{align*}
y = 0 & \rightarrow \frac{5}{3}x + 0 = 4 \\
& \quad \frac{5}{3}x = 4 \\
& \quad x = 2.4 \\
& \quad \quad (2.4, 0) \\
\text{Put both the number we chose for x or y and the number we solved for into an ordered pair (x,y)}
\end{align*}\]

\[\begin{align*}
y = -5 & \rightarrow \frac{5}{3}x + (-5) = 4 \\
& \quad \frac{5}{3}x = 9 \\
& \quad x = 5.4 \\
& \quad \quad (5.4, -5)
\end{align*}\]
Now that we can get so many solutions, the best way to represent them is with a graph. If we plot the pairs that we found for the previous equation we used \((3x + 2y = 5)\) we get this:

\((3,-2), (1,1), (-1,4), (5,-5), (7,-8)\)

You will notice that all of the solutions are in a straight line. It is important to realize that if we draw the line that connects the ordered pairs, all of the points on that line are solutions. The problems will simply ask you to graph the line \(3x+2y=5\) or something similar.

The correct answer to “Graph the line \(3x+2y=5\)”, is then the graph at the left.

To organize the points we make, we can outline the points on the graph by using a table:

\[
y = \frac{1}{4}x - 2
\]

when \(x = 4\) we have \(y = \frac{1}{4}(4) - 2\) which means \(y = -1\).
when \(x = 4\) we have \(y = \frac{1}{4}(0) - 2\) which means \(y = -2\).
when \(y = 0\) we have \(0 = \frac{1}{4}(x) - 2\) which means \(x = 8\).
when \(y = 3\) we have \(3 = \frac{1}{4}(x) - 2\) which means \(x = 20\).

**EXAMPLES**

3. **Give a table of solutions for the equation and graph: \(y = \frac{2}{3}x - 2\)**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(\frac{2}{3}(3) - 2)</td>
</tr>
<tr>
<td>0</td>
<td>(\frac{2}{3}(0) - 2)</td>
</tr>
<tr>
<td>-3</td>
<td>(\frac{2}{3}(-3) - 2)</td>
</tr>
</tbody>
</table>

Pick a few points for \(x\) or \(y\) and solve for the other variable.
Put both the \(x\) and \(y\) values into a table

Plot the points that we just put into our table on the graph with dots

Draw a line connecting the dots
**Give a table of solutions for the equation and graph:** \( y = 1.5x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( y = 1.5(-4) + 3 \rightarrow y = -3 )</td>
</tr>
<tr>
<td>-2</td>
<td>( y = 1.5(-2) + 3 \rightarrow y = 0 )</td>
</tr>
<tr>
<td>0</td>
<td>( y = 1.5(0) + 3 \rightarrow y = 3 )</td>
</tr>
</tbody>
</table>

Pick a few points for \( x \) or \( y \) and solve for the other variable.

Put both the \( x \) and \( y \) values into a table.

Plot the points that we just put into our table on the graph with dots.

Draw a line connecting the dots.

**GRAPHING USING X- AND Y- INTERCEPTS**

**LAWS & PROCESSES**

Remember the “Pick & Stick” method? Well, when picking things to use, the easiest ones are usually when \( x = 0 \) or when \( y = 0 \). You’ll notice that when \( x = 0 \) the point is on the \( y \)-axis. Likewise, when \( y = 0 \) the point is on the \( x \)-axis. In the previous example, the point (0,-2) lies on the \( y \)-axis and is called the **y-intercept**; the point (8,0) lies on the \( x \)-axis and is called the **x-intercept**.

**An x-intercept happens when y is zero, and a y-intercept happens when x is zero**
5. Find the x and y intercepts and graph: \(4x + 3y = 8\)

\[4x + 3(0) = 8 \quad \rightarrow x = 2\]

\[4(0) + 3y = 8 \quad \rightarrow y = \frac{8}{3}\]

\(\begin{array}{|c|c|}
\hline
x & y \\
\hline
2 & 0 \\
0 & \frac{8}{3} \\
\hline
\end{array}\)

To find the x-intercept plug in 0 for y in the equation and solve.

To find the y-intercept plug in 0 for x in the equation and solve.

Plot the points that we just put into our table on the graph with dots.

Draw a line connecting the dots.

5. Find the x and y intercepts and graph: \(y = -2x + 1\)

\[0 = -2x + 1 \quad \rightarrow x = .5\]

\[y = -2(0) + 1 \quad \rightarrow y = 1\]

\(\begin{array}{|c|c|}
\hline
x & y \\
\hline
.5 & 0 \\
0 & 1 \\
\hline
\end{array}\)

To find the x-intercept plug in 0 for y in the equation and solve.

To find the y-intercept plug in 0 for x in the equation and solve.

Plot the points that we just put into our table on the graph with dots.

Draw a line connecting the dots.
There are a couple of particular kinds of lines that may give you a bit of trouble when you first see them. For example: $x = 4$ is the equation of a line.

What is $x$ when $y$ is 4? Answer: 4
What is $x$ when $y$ is 0? Answer: 4
What is $x$ when $y$ is -3? Answer: 4

Since $y$ is not in the equation, $y$ can be anything it wants to be, but $x$ is always 4. The graph is as follows:

Here is the line $x = 4$; notice that it is vertical and hits where $x$ is 4.

The other special case, although it may look difficult at first, is very similar to the previous example. Let's look at $y = -2$:

What is $y$ when $x$ is 0? Answer: -2
What is $y$ when $x$ is 5? Answer: -2
What is $y$ when $x$ is -3? Answer: -2

Here is the line $y = -2$; notice that it is horizontal and hits where $y$ is -2.
Graph the line: \( x = -1 \)

\[
\begin{align*}
y &= 1 \Rightarrow x = -1 \\
y &= -1 \Rightarrow x = -1
\end{align*}
\]

If you don’t remember if it is a horizontal line or a vertical line, pick a few \( y \) values and see what \( x \) equals.

Plot the points that we just came up with on the graph with dots.

Draw a line connecting the dots.

Graph the line: \( 2 = y \)

\[
\begin{align*}
x &= 1 \Rightarrow y = 2 \\
x &= -1 \Rightarrow y = 2
\end{align*}
\]

If you don’t remember if it is a horizontal line or a vertical line, pick a few \( x \) values and see what \( y \) equals.

Plot the points that we just came up with on the graph with dots.

Draw a line connecting the dots.

Section 3.1
1. Three types of bears are in a national park. The number of grizzly bears are 4 more than twice the number of black bears, and 50 more panda bears than black bears. There are a total of 874 bears in the park. How many of each kind are there?

2. An international phone call costs 35¢ to connect and 12¢ for every minute of the call. How long can a person talk for $3.60?

3. A 52m rope is cut so that one piece is 18m longer than the other. What are the lengths of the pieces?

4. Original Price: $292.50
   Discount: 20%
   Final Price: 

5. Original Price:
   Discount: 40%
   Final Price: $73.90

Solve and graph.

6. $5(x-2) > 7x + 8$

3.1 Fill out the table for each of the following:

7. $x + y = 9$
8. $2x - y = 5$
9. $5x + 4y = 9$
10. $x - 7y = 13$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-4</td>
<td>2</td>
<td>-3</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph the following lines, and label three points.

11. $3x + y = 10$
12. $y = 2x$
13. $x - 4y = 7$
14. $x = 3$
15. $y = -\frac{3}{2}x + 4$
16. $6x - 5y = 12$
17. $y = -4$
18. $5x + 2y = 6$

Preparation.

19. After reading a bit of section 3.2, try to find the slope between (4,1) and (7,11).
Answers:
1. 205 Black, 414 Grizzly, 255 Panda
2. 27 minutes
3. 17m, 35m
4. $234
5. $123.17
6. $x < -9$
7. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>-4</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>
8. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>-7</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
9. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>-15</td>
<td>5</td>
</tr>
</tbody>
</table>
10. 
   | x | y |
    |---|---|
    | 20 | 1 |
    | 34 | 3 |
    | 2 | 11 |
    | 0 | 13 |
    | 6 | -1 |
11. (0,10) (3,1) (-1,13)
12. (0,0) (1,2) (2,4)
13. (7,0) (3,-1) (0,-7/4)
14. (3,0) (3,1) (3,2)
15. (0,4) (7,1) (14,-2)
16. (2,0) (0,-12/5) (7,6)
17. (0,-2) (2,-4) (37,-4)
18. (0,3) (2,-2) (6/5,0)
19. $m = \frac{10}{3}$
3.2 Slope

OBJECTIVES
- Understand slope and how to calculate it
- Know the difference between slope = zero and undefined slope

A LINES (Overview)

CHAPTER 3 TOPICS

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<th>y vs. x</th>
<th>Solution Sets</th>
<th>Graphs</th>
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<td>$m = \frac{y_1 - y_2}{x_1 - x_2}$</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

EXAMPLE

Graph: $3x + 2y = 4$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$m = \frac{2 - (-1)}{0 - 2}$

$m = -\frac{3}{2}$
**SLOPE**: This refers to the steepness of a line as it goes up or down as observed left to right.

- Slope is typically represented by the letter “m”
- It is also referred to as the rate of change

The slope of the line between point A(-2,1) and B(3,3) is found by taking how much it changes up and down (distance between 1 and 3 = 2) over how much it changes left and right (distance between -2 and 3 = 5). That makes a slope of $\frac{2}{5}$. We could say that “m = $\frac{2}{5}$”.

\[ \text{Slope} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} \]

There are some trends that you will observe as we calculate slope:

**Trends with Slope**

1. Bigger numbers for slope correspond to steeper lines.
2. Positive slopes head up as you go to the right.
3. (Opposite of #2) Negative slopes will head down as you go to the right.
To get a feel for slope a little bit better, look at some of these slopes. You will notice that the higher the number, the steeper it is. On the other hand, numbers get increasingly large in the negative direction for lines that are heading down ever steeper.

\[ m = \text{undefined} \]

\[ m = 15 \]

\[ m = 2 \]

\[ m = 1 \]

\[ m = \frac{1}{2} \]

\[ m = \frac{1}{3} \]

\[ m = 0 \]

\[ m = -\frac{1}{3} \]

\[ m = -\frac{2}{3} \]

\[ m = -1 \]

\[ m = -2 \]

\[ m = \text{undefined} \]

Sometimes, people refer to lines as having “no slope.” Since this can be misleading (does that mean straight up or horizontal), we will avoid the term all together:

A vertical line has undefined slope and a horizontal line has a slope of zero.

We will discuss why this is in the next subsection.

\[ m = \text{undefined} \]

\[ m = 0 \]

\[ m = -\frac{1}{2} \]

\[ m = \frac{1}{3} \]

\[ m = -1 \]

\[ m = -2 \]

\[ m = \text{undefined} \]

**COMMON MISTAKES**

A vertical line has undefined slope and a horizontal line has a slope of zero.

**LAWS & PROCESSES**

In the first example we obtained the 2 as the distance from 1 to 3. What operation finds distance? **Answer: Subtraction.** Using subtraction, we find slopes a little more quickly. Let’s look at those three examples, using subtraction this time:

<table>
<thead>
<tr>
<th>1st Example:</th>
<th>2nd Example:</th>
<th>3rd Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{3 - 1}{3 - (-2)} = \frac{2}{5} ]</td>
<td>[ \frac{7 - 1}{1 - (-2)} = 2 ]</td>
<td>[ \frac{-3 - 1}{5 - (-2)} = -\frac{4}{7} ]</td>
</tr>
<tr>
<td>Points: (3,3) &amp; (−2,1)</td>
<td>(1,7) &amp; (−2,1)</td>
<td>(5, −3) &amp; (−2,1)</td>
</tr>
</tbody>
</table>

Section 3.2
If we follow this pattern using ‘x’ and ‘y’, we make a formula for slope. We will call point #1 
\((x_1,y_1)\) showing that the x and the y come from the 1st point. Similarly we will call point #2 \((x_2,y_2)\). 
Now you can find the slope just like we did in the previous examples:

![Formula for Slope]

\[
m = \frac{y_1 - y_2}{x_1 - x_2}
\]

Sometimes this formula is written in a few different ways. Here are some of the others:

\[
m = \frac{\text{y change}}{\text{x change}} \quad m = \frac{\text{rise}}{\text{run}} \quad m = \frac{\Delta y}{\Delta x}
\]

They all mean the same thing.

Remember how we said that “vertical lines have undefined slope and horizontal lines have zero slope”?

We will work two examples of this:

**First: \(y = -2\)**

For this graph, let’s use these points: \((3,-2)\) and \((-2,-2)\). It gives us the graph 
of a horizontal line where \(y = -2\). Putting those two points in to the formula 
for finding slope, we get:

\[
m = \frac{-2 - (-2)}{3 - (-2)} = \frac{0}{5} = 0
\]

which means that **all horizontal lines will have a slope of 0**.

**Second: \(x=2\)**

Use these points: \((2,-2)\) and \((2,3)\). It gives us the graph of a vertical line 
where \(x = 2\). Now if we put the points in the slope formula, we get:

\[
m = \frac{-2 - 3}{2 - 2} = \frac{-5}{0} = \text{UNDEFINED}
\]

(Division by zero is undefined.)

This means that all **vertical lines have undefined slope**.
Calculate the slope of the line formed by the following points:

1. (2, 10) and (−1, 1)

\[ m = \frac{10 - 1}{2 - (-1)} = \frac{9}{3} = 3 \]

Answer: The slope is 3

2. (−3, 2) and (0, −2)

\[ m = \frac{2 - (-2)}{(-3) - 0} = \frac{4}{-3} = -\frac{4}{3} \]

Answer: The slope is $-\frac{4}{3}$
1. Three types of trees are in a local park. There were 5 more than three times as many aspens as oaks, and 20 less maples than oaks. There are a total of 850 trees in the park. How many of each kind are there?

2. If the length is 3 more than 4 times the width of a rectangle and the perimeter is 76mm, what are the dimensions?

3. If grading in a class is set up so that 10% is attendance, 10% is tutoring, 30% homework, and 50% tests, what is a student’s grade if he has 80% attendance, 50% tutoring, 50% homework and 80% on tests?

4. Original Price: $392.50
   Discount: 20%
   Final Price: 

5. Original Price: 
   Discount: 45%
   Final Price: $73.90

6. Four consecutive odd integers add up to 328. What are the four numbers?

7. Fill out the table for each of the following:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

8. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

9. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

10. | x | y |
    |---|---|
    | 2 |   |
    | 5 |   |
    | 2 |   |
    | 0 |   |
    | -1|   |

11. 3x + 2y = 10
12. y = 2x - 7
13. y = \(\frac{1}{2}x\)
14. x = -6
15. y = -\(\frac{3}{7}x - 2\)
16. 2x - 5y = 12
17. y = 5
18. 5x + y = 6

19. (5, -2) (7,3)
20. (4,1) (-5,6)
21. (5, -1) (-3, -8)
22. (7,3) (-2,3)
23. (-5,2) (4, -3)
24. (-6,1) (-6,5)

25. Explain the difference between a slope of zero and an undefined slope.

Preparation

26. Find two points of each line and then use those points to find the slope.

   \(2x - 3y = 1\) \hspace{2cm} \(y = \frac{3}{5}x + 4\)
Answers
1. 173 Oaks, 524 Aspen, 153 Maple
2. \( w = 7 \text{mm}, l = 31 \text{mm} \)
3. 68%
4. $314
5. $134.36
6. 79, 81, 83, 85
7. \[
\begin{array}{c|c}
    x & y \\
    \hline
    5 & -1 \\
    -4 & 17 \\
    3 & 3 \\
    \frac{9}{5} & 0 \\
    1 & 7 \\
\end{array}
\]
8. \[
\begin{array}{c|c}
    x & y \\
    \hline
    2 & 12 \\
    0 & 2 \\
    -1 & -3 \\
    \frac{2}{5} & 0 \\
    \frac{2}{5} & 4 \\
\end{array}
\]
9. \[
\begin{array}{c|c}
    x & y \\
    \hline
    1 & 2 \\
    0 & \frac{9}{5} \\
    -3 & 3 \\
    9 & 0 \\
    -11 & 5 \\
\end{array}
\]
10. \[
\begin{array}{c|c}
    x & y \\
    \hline
    35 & 2 \\
    42 & 5 \\
    2 & \frac{85}{7} \\
    0 & -13 \\
    28 & -1 \\
\end{array}
\]
11. \((0,5), \left(\frac{10}{2}, 0\right), (2,2)\)
12. \((-7,0), (1,-5), (2,-3)\)
13. \((0,0), (2,1), (8,4)\)
14. \((-6,0), (-6,1), (-6,2)\)
15. \((0,-2), (7,-5), (-7,1)\)
16. \((6,0), \left(0, -\frac{12}{5}\right), (1,-2)\)
17. \((0,5), (-2,5), (3,5)\)
18. \((0,6), (1,1), \left(\frac{6}{5}, 0\right)\)
19. \(m = \frac{5}{2}\)
20. \(m = -\frac{5}{9}\)
21. \(m = \frac{7}{8}\)
22. \(m = 0\)
23. \(m = -\frac{5}{9}\)
24. \(m = \text{undefined}\)
25. Undefined is vertical
Straight up and down
0 is horizontal
Straight across
26. In class

Section 3.2
3.3 Graphing with Slope

**OBJECTIVES**
- Find slope from the standard form of a line
- Use slope to graph lines
- Find slope from slope-intercept form of a line
- Use slope and y-intercept to graph lines

**CHAPTER 3 TOPICS**

<table>
<thead>
<tr>
<th>Graphing</th>
<th>y vs. x</th>
<th>Solution Sets</th>
<th>Graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>$m = \frac{y_1 - y_2}{x_1 - x_2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Graphing with Slope:**
- **Standard Form**
  - $Ax + By = C$

**Graphing with Slope:**
- **Slope-Intercept Form**
  - $y = mx + b$

**Writing Equations**
- Translation

**Parallel and Perpendicular Slopes**
- Parallel = Same
- Perpendicular = Negative

**Line Inequalities**

**EXAMPLES**

$-2x + y = -4$

$m = -\frac{A}{B} = -\frac{-2}{1} = 2$

$y = 0, \quad \therefore -2x = -4$

$\therefore x = 2$

$m = 2$ and point at (2,0)
SLOPE (Section 3.2):

\[ m = \frac{y_1 - y_2}{x_1 - x_2} \quad m = \frac{\text{ychange}}{\text{xchange}} \quad m = \frac{\text{rise}}{\text{run}} \quad m = \frac{\Delta y}{\Delta x} \]

**STANDARD FORM:** There are two ways to approach writing an equation for a line. The first way is written in this format:

\[ Ax + By = C \]

A, B, and C are usually integers in this case.

**SLOPE-INTERCEPT FORM:** The second way to write the equation of a line consists of putting ‘y’ equal to everything else. You can get the slope-intercept form from the standard form by just solving for y.

\[ y = mx + b \]

‘m’ in this equation IS SLOPE. ‘b’ is the “y-intercept” because when x = 0, then y = b:

\[ y = m(0) + b \]

\[ \therefore y = b \]

If you practiced finding slope by picking points, then solving, for a few equations in standard form, you might see something like this:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope:</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x – 5y = 10</td>
<td>( m = \frac{3}{5} )</td>
</tr>
<tr>
<td>2x + 9y = 4</td>
<td>( m = -\frac{2}{9} )</td>
</tr>
<tr>
<td>5x + y = 15</td>
<td>( m = -5 )</td>
</tr>
<tr>
<td>x-3y = 12</td>
<td>( m = \frac{1}{3} )</td>
</tr>
</tbody>
</table>

If you noticed, there’s a pattern emerging that you would be able to use as a shortcut.

Do you see how the change in y is always the coefficient of x?

Do you see that the change in x is always the opposite of the coefficient of y?

For standard form, this is always the case. This means that:

\[ Ax + By = C \]

\[ m = -\frac{A}{B} \]
Calculate the slope for the equation:

\[ 4x - 7y = 14 \]

\[
m = -\frac{4}{-7}
\]

Plug in our values for A and B into our equation for slope in standard form

Simplify the equation

\[ m = \frac{4}{7} \]

Answer: The slope is \( \frac{4}{7} \).

Calculate the slope for the following equation:

\[ 14x + 2y = 3.5 \]

\[
m = -\frac{14}{2}
\]

Plug in our values for A and B into our equation for slope in standard form

Simplify the equation

\[ m = -\frac{14}{2} = -7 \]

Answer: The slope is -7.

Graphing a line by calculating all the points and then “connecting the dots” can be time consuming – we don’t want to have to do it for long. Graphing a line with slope is much faster.

If slope is really just the change in ‘y’ over change in ‘x,’ then all we need to do is find one point, follow the slope to the next point, then connect the two dots.
Calculate the slope and graph the line:

\[ 2x - 3y = 9 \]

Since the equation is in standard form, we can calculate slope using the formula \( m = -\frac{A}{B} \)

\[ m = \frac{2}{3} \]

We can solve for a point by just putting in zero for ‘y’ or ‘x’. We’ll replace ‘x’ here.

\[ 2(0) - 3y = 9 \]
\[ -3y = 9 \]
\[ y = -3 \]

There is a point at \((0, -3)\)

Now, the slope says that ‘y’ changes by +2, and ‘x’ changes by 3. So we go up 2 and over 3 and make the new dot.

Now all we have to do is connect the dots.
4 Calculate the slope and graph the line:

\[ 4x - 3y = 8 \]

Since the equation is in standard form, we can calculate slope using the formula \( m = -\frac{A}{B} \).

We can solve for a point by just putting in zero for ‘y’ or ‘x’. We’ll replace ‘y’ here.

Now, the slope says that ‘y’ changes by +4 and ‘x’ changes by 3. So we go up 4 and over 3 and make the new dot.

Now all we have to do is connect the dots.

There is a point at (2,0)
Getting slope from the Slope-Intercept form is so easy – you just have to look. Since the form of the line is \( y = mx + b \), and \( m \) is the slope, just look at the coefficient of \( x \) and you have the slope.

Since this is so easy to recognize, it is often the preferred set up for an equation. However, both forms have advantages and disadvantages. You need to be able to convert from one to the other.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -2x - 5 )</td>
<td>( m = -2 )</td>
</tr>
<tr>
<td>( y = \frac{1}{7}x + 4 )</td>
<td>( m = \frac{1}{7} )</td>
</tr>
<tr>
<td>( y = -\frac{4}{9}x - 13 )</td>
<td>( m = -\frac{4}{9} )</td>
</tr>
<tr>
<td>( y = \frac{7}{9} - 2 )</td>
<td>( m = \frac{1}{7} )</td>
</tr>
</tbody>
</table>

Since this is so easy to recognize, it is often the preferred set up for an equation. However, both forms have advantages and disadvantages. You need to be able to convert from one to the other.

### EXAMPLES

#### 5
Convert this equation into slope-intercept form:
\[ 3x + 4y = 8 \]

Get the y’s alone on one side

\[ 3x + 4y = 8 \]
\[ -3x \]
\[ 4y = -3x + 8 \]
\[ 4y = -3x + 8 \]
\[ \div 4 \]
\[ y = -\frac{3}{4}x + 2 \]

Divide every term by the number attached to the y

#### 6
Convert this equation into slope-intercept form:
\[ x - 2y = 3 \]

Get the y’s alone on one side

\[ x - 2y = 3 \]
\[ -x \]
\[ -2y = -x + 3 \]
\[ -2y = -x + 3 \]
\[ \div (-2) \]
\[ \frac{1}{2}x - \frac{3}{2} \]

Divide every term by the number attached to the y
We can graph any line if we have a slope and a point. As we have already talked about, we can find the slope from slope-intercept form pretty easily. Since \( y = b \) when \( x = 0 \), the starting point can always be \((0, b)\).

**EXAMPLES**

**Graph the equation:**

\[
y = \frac{1}{3}x - 2
\]

\[m = \frac{1}{3}, \quad y \text{ intercept } = (0, -2)\]

Determine the slope and the y intercept from the equation

Plot the y intercept

Trace out the next point by following the slope. In this example go up 1 and over 3

Connect the two dots we just made
Graph the equation:

\[ y = -4x + 3 \]

\[ m = -4 \]
\[ y \text{ intercept} = (0,3) \]

Determine the slope and the y-intercept from the equation.

Plot the y-intercept.

Trace out the next point by following the slope. In this example go down 4 and over 1.

Connect the two dots we just made.
1. Three types of horses are in a local ranch. There are 8 more than twice as many Arabians as Quarter-horses, and 50 more Clydesdales than Quarter-horses. There are a total of 282 horses at the ranch. How many of each kind are there?

2. What is the radius of a cone that has Lateral Surface Area of 197.92 in² and a slant height of 9 in?

3. Solve and graph the solution: \(3x - 1 > \frac{5}{2} x + 9\)

4. Original Price:$392.50
   Tax: 6%
   Final Price:

5. Original Price:
   Tax: 7%
   Final Price: $73.90

Fill out the table for each of the following:

6. \(2x - 3y = 9\)

7. \(y = \frac{7}{2}x + 2\)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Graph the following lines, and label x and y intercepts.

8. \(5x + 2y = 10\)

9. \(y = \frac{3}{7}x - 6\)

10. \(y = \frac{8}{3}x\)

11. \(x = 10\)

12. \(y = -\frac{3}{7}x + 4\)

13. \(7x - y = 14\)

Find the slope between each pair of points.

14. \((8,-2) (7,3)\)

15. \((8,1) (-5,6)\)

16. \((-3,-1) (-3,-8)\)

17. \((7,9) (-2,3)\)

18. \((-5,2) (4,6)\)

19. \((-6,1) (6,1)\)

Graph the following lines giving one point and the slope.

20. \(-6x + y = 10\)

21. \(y = 4x + 3\)

22. \(y = \frac{1}{2}x - 4\)

23. \(x = -6\)

24. \(y = -\frac{3}{7}x - 2\)

25. \(3x - 4y = 12\)

26. \(5x + 3y = 10\)

27. \(x + 4y = 9\)

28. \(y = 7\)

Preparation.

29. Fill out the Slope Monster (on the next page)

30. Write down 5 equations of lines that have the slope: \(m= -\frac{3}{8}\)
## Slope Monster

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Equation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 5y = 7$</td>
<td>$y = \frac{5}{9}x - 4$</td>
<td>$4x - y = 7$</td>
<td>$y = \frac{5}{2}x - 4$</td>
</tr>
<tr>
<td>$5x - 3y = 7$</td>
<td>$8x - 3y = 12$</td>
<td>$2x + 7y = 19$</td>
<td>$-4x + 7y = 19$</td>
</tr>
<tr>
<td>$2x + 7y = 19$</td>
<td>$x = 13$</td>
<td>$x = -19$</td>
<td>$y = \frac{8}{7}x - 4$</td>
</tr>
<tr>
<td>$3x - 8$</td>
<td>$y = \frac{8}{3}x - 8$</td>
<td>$y = -3x - 8$</td>
<td>$y = \frac{8}{3}x - 4$</td>
</tr>
<tr>
<td>$-3x + 9y = 4$</td>
<td>$y = -3$</td>
<td>$-10x + 6y = 4$</td>
<td>$y = -3x - 8$</td>
</tr>
<tr>
<td>$y = -\frac{3}{11}x - 4$</td>
<td>$y = \frac{6}{7}x - 4$</td>
<td>$7x - 3y = 7$</td>
<td>$2x - 8y = 17$</td>
</tr>
<tr>
<td>$5x - 3y = 7$</td>
<td>$y = \frac{2}{9}x - 4$</td>
<td>$y = \frac{5}{2}x + 6$</td>
<td>$y = \frac{6}{11}x - 4$</td>
</tr>
<tr>
<td>$4x + 7y = 19$</td>
<td>$x = -3$</td>
<td>$x = 7$</td>
<td>$y = \frac{2}{9}x - 4$</td>
</tr>
<tr>
<td>$x = 3$</td>
<td>$y = -\frac{8}{5}x - 4$</td>
<td>$y = \frac{5}{3}x - 4$</td>
<td>$y = \frac{5}{2}x - 4$</td>
</tr>
<tr>
<td>$y = -2x - 8$</td>
<td>$y = 4x + 13$</td>
<td>$y = 4x - 13$</td>
<td>$y = -3x - 6y = 4$</td>
</tr>
<tr>
<td>$-3x + 6y = 4$</td>
<td>$y = -5$</td>
<td>$y = 7$</td>
<td>$y = -5$</td>
</tr>
<tr>
<td>$y = -\frac{3}{4}x - 4$</td>
<td>$y = -\frac{3}{4}x - 4$</td>
<td>$y = -\frac{3}{5}x + 15$</td>
<td>$y = -\frac{3}{5}x + 15$</td>
</tr>
</tbody>
</table>
Answers:
1. 56 Quarter-horses, 106 Clydesdales, 120 Arabian
2. radius = 7 in
3. $x > 20$
4. $\$416.05$
5. $\$69.07$
6. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   5 & \frac{1}{3} \\
   -4 & -\frac{17}{3} \\
   9 & 3 \\
   \frac{9}{2} & 0 \\
   15 & 7 \\
   \end{array}
   \]
7. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   2 & 9 \\
   0 & 2 \\
   -1 & -\frac{3}{2} \\
   -\frac{4}{7} & 0 \\
   \frac{4}{7} & 4 \\
   \end{array}
   \]
8. $(0.5, 2.0)$
9. $(0.6, \frac{21}{2}, 0)$
10. $(0, 0)$
11. $(10, 0)$ no y-int
12. $(0, 4) \left( \frac{28}{3}, 0 \right)$
13. $(2, 0) (0, -14)$
14. $m = -5$
15. $m = -\frac{5}{13}$
16. $m$ undefined
17. $m = \frac{2}{3}$
18. $m = \frac{4}{7}$
19. $m = 0$
20. $(0, 10); m = 6$
21. $(0, 3); m = 4$
22. $(0, -4); m = \frac{1}{2}$
23. $(-6, 0); m$ undefined
24. $(0, -2); m = -\frac{3}{7}$
25. \((4,0); m = \frac{3}{4}\)

26. \((2,0); m = -\frac{5}{3}\)

27. \((9,0); m = -\frac{1}{4}\)

28. \((15,7); m = 0\)

29. Correct it in class.

30. In class.
### Question
Write the equation of the line that has a slope: \(\frac{3}{5}\) and goes through the point: \((5, -2)\) and then graph the line.

### Solution
\[
y = \frac{3}{5}x - 5
\]
Since we can derive information, such as slope, from an equation, it isn’t that difficult to write an equation if we have the slope. Being able to do this becomes quite significant, as you will see in the next subsection.

There are really two kinds of set-ups for a basic translation problem from words to an equation: either you are given two points, or you are given one point and the slope. Since you already know how to calculate slope from two points, the first shouldn’t be much more difficult than the second. We will approach each one, solving for an equation in standard form, and then in slope-intercept form.

### Given Two Points

1. Use the points to find the slope
2. If asked to write in **standard form:**
   a. Remember that \( m = \frac{A}{B} \). Use the slope to fill in A & B in \( Ax + By = C \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for \( C \)
   d. Re-write with only \( x \) and \( y \) as variables
3. If asked to write in **slope-intercept form:**
   a. Fill in the slope for \( m \) in \( y = mx + b \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for \( b \)
   d. Re-write with only \( x \) and \( y \) as variables

### Given Slope and One Point

Follow the same process as if you were given two points, but now you don’t have to find the slope.
Write the equation of a line that goes through the points: (-2, -2) and (2, 4) and present it in standard form. Graph the equation.

Calculate the slope from the two given points
Remember: \( m = \frac{y_1 - y_2}{x_1 - x_2} \)

Evaluate: 
\[
\begin{align*}
\frac{-2 - 4}{-2 - 2} &= \frac{-6}{-4} \\
&= \frac{3}{2}
\end{align*}
\]

Remember that \( \frac{-A}{B} = m \)

So if \( \frac{A}{B} = \frac{3}{2} \), then \( A = 3 \) and \( B = -2 \)

\[
(3)(2) + (-2)(4) = C \\
-2 = C
\]

\[
3x - 2y = -2
\]

Plug A, B, and the x and y from a point we are given into the standard form equation to solve for C
Plug in our A, B, and C into the standard form equation
Now graph the equation by plotting the two points we have been given and connecting the dots

---

Write the equation of a line that goes through the point (-3, 4) and has a slope of \( \frac{-2}{3} \). Present it in slope-intercept form and graph the equation.

Plug the point we have been given and the slope into the slope-intercept equation.

Solve for b
Write out the final equation with the slope and with the y-intercept filled in
Graph the equation using the y-intercept and then counting out the slope
Section 3.4

3.4 EXERCISE SET

Fill out the table for each of the following:

<table>
<thead>
<tr>
<th></th>
<th>1. $2x - 5y = 11$</th>
<th>2. $y = \frac{7}{2}x + 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph the following lines, and label x and y intercepts.

3. $4x - 2y = 10$
4. $y = -\frac{5}{3}x - 6$
5. $y = 5x$

Find the slope between each pair of points.

6. (3,-2) (7,3)
7. (9,1) (-7,6)
8. (5,-1) (-3,-8)
9. (-2,9) (-2,3)
10. (-5,2) (5,6)
11. (19,1) (6,1)

12. Explain the difference between a slope of zero and an undefined slope.

Graph the following lines giving one point and the slope.

13. $-3x + 4y = 10$
14. $y = 2x - 7$
15. $y = \frac{2}{5}x - 4$
16. $y = 17$
17. $y = -\frac{3}{7}x - 2$
18. $2x - 6y = 12$

Write the equations of the lines with the slopes and points:

Ex. Write an equation of the line that has slope $m = \frac{4}{7}$, and goes through the point (2,1). Put the answer in Standard Form.

From the slope $m = \frac{4}{7}$, I know that the equation must look like:

$4x - 7y = \text{something}$, so I put in the point to see what it is.

$4(2) - 7(1) = 1$.

Thus the answer is $4x - 7y = 1$. 
Ex. Write an equation of the line that has slope \( m = \frac{7}{4} \), and goes through the point (2,1). Put the answer in Slope-Intercept Form.

From the slope \( m = \frac{7}{4} \), I know that the equation must look like:

\[
y = \frac{7}{4}x + b
\]

Put the point in to see what \( b \) is.

\[
1 = \frac{7}{4}(2) + b
\]

\[
1 - \frac{7}{2} = b
\]

\[
-\frac{1}{2} = b
\]

Thus the answer is \( y = \frac{7}{4}x - \frac{1}{2} \).

19. Write an equation of the line that has slope \( m = -3 \), and goes through the point (-4,6). Put the answer in Standard Form.

20. Write an equation of the line that has slope \( m = \frac{3}{8} \), and goes through the point (3,6). Put the answer in Standard Form.

21. Write an equation of the line that has slope \( m = -\frac{3}{4} \), and goes through the point (1,-3). Put the answer in Slope-Intercept Form.

22. Write an equation of the line that has slope \( m = -\frac{4}{5} \), and goes through the point (5,-3). Put the answer in Slope-Intercept Form.

23. Write an equation of the line that has slope \( m = 2 \), and goes through the point (0,5). Put the answer in Slope-Intercept Form.

24. Write an equation of the line that has slope \( m = -\frac{1}{7} \), and goes through the point (-4,7). Put the answer in Standard Form.

25. Fill out the Slope Monster (on the next page). Record the time it takes you to complete it.

Preparation

26. What are the similarities between the graphs of the following lines?

\[
2x - 3y = 3 \quad \text{and} \quad 3x + 2y = 2
\]

\[
y = 2x - 5 \quad \text{and} \quad y = 2x + 7
\]
## Slope Monster

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>Equation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 5y = 7$</td>
<td>$y = \frac{5}{9}x - 4$</td>
<td>$4x - y = 7$</td>
<td>$y = \frac{5}{2}x - 4$</td>
</tr>
<tr>
<td>$5x - 3y = 7$</td>
<td>$x = 13$</td>
<td>$8x - 3y = 12$</td>
<td>$x = -19$</td>
</tr>
<tr>
<td>$2x + 7y = 19$</td>
<td>$y = \frac{8}{3}x - 8$</td>
<td>$- 4x + 7y = 19$</td>
<td>$y = \frac{8}{7}x - 4$</td>
</tr>
<tr>
<td>$5x - 3y = 7$</td>
<td>$y = \frac{2}{9}x - 4$</td>
<td>$4x + 7y = 7$</td>
<td>$y = -3x - 8$</td>
</tr>
<tr>
<td>$-3x + 9y = 4$</td>
<td>$y = -3$</td>
<td>$-10x + 6y = 4$</td>
<td>$y = 15$</td>
</tr>
<tr>
<td>$y = -\frac{3}{11}x - 4$</td>
<td>$y = \frac{6}{11}x - 4$</td>
<td>$2x - 8y = 17$</td>
<td>$y = \frac{5}{2}x + 6$</td>
</tr>
<tr>
<td>$7x - 3y = 7$</td>
<td>$y = \frac{2}{9}x - 4$</td>
<td>$4x + 7y = 7$</td>
<td>$x = - 3$</td>
</tr>
<tr>
<td>$2x - 9y = 19$</td>
<td>$y = \frac{5}{2}x + 6$</td>
<td>$2x - 9y = 19$</td>
<td>$x = 7$</td>
</tr>
<tr>
<td>$x = - 3$</td>
<td>$y = \frac{5}{3}x - 4$</td>
<td>$y = \frac{5}{3}x - 4$</td>
<td>$y = \frac{5}{3}x - 4$</td>
</tr>
<tr>
<td>$y = -3x - 8$</td>
<td>$y = 4x + 13$</td>
<td>$y = 4x + 13$</td>
<td>$y = 4x + 13$</td>
</tr>
<tr>
<td>$-3x + 6y = 4$</td>
<td>$y = -5$</td>
<td>$-3x - 6y = 4$</td>
<td>$y = -3x - 8$</td>
</tr>
<tr>
<td>$y = -2x - 8$</td>
<td>$y = 7$</td>
<td>$y = 7$</td>
<td>$y = 7$</td>
</tr>
<tr>
<td>$y = -\frac{3}{4}x - 4$</td>
<td>$y = \frac{3}{5}x + 15$</td>
<td>$y = \frac{3}{5}x + 15$</td>
<td>$y = \frac{3}{5}x + 15$</td>
</tr>
</tbody>
</table>
Answers:

1. \( \begin{array}{c|c}
   x & y \\
   \hline
   5 & -\frac{1}{5} \\
   -4 & -\frac{19}{5} \\
   13 & 3 \\
   \frac{11}{2} & 0 \\
   23 & 7 \\
\end{array} \)

2. \( \begin{array}{c|c}
   x & y \\
   \hline
   2 & 13 \\
   0 & 6 \\
   -1 & \frac{5}{2} \\
   -\frac{12}{7} & 0 \\
   -\frac{4}{7} & 4 \\
\end{array} \)

3. \((0,-5) (\frac{6}{2},0)\)

4. \((0,-6) (\frac{18}{5},0)\)

5. \((0,0) (1,5)\)

6. \(m = \frac{5}{4}\)

7. \(m = -\frac{5}{16}\)

8. \(m = \frac{7}{8}\)

9. \(m = \text{undefined}\)

10. \(m = \frac{2}{5}\)

11. \(m = 0\)

12. Undefined is vertical
   0 is horizontal

13. \((0,\frac{5}{2}) m = \frac{3}{4}\)

14. \((0,-7) m = 2\)

15. \((0,-4) m = \frac{2}{5}\)

16. \((0,17) m = 0\)

17. \((0,-2) m = -\frac{3}{2}\)

18. \((6,0) m = \frac{1}{3}\)

19. \(3x + y = -6\)

20. \(5x - 8y = -33\)

21. \(y = -\frac{2}{3}x - \frac{7}{3}\)

22. \(y = -\frac{5}{6}x + 1\)

23. \(y = 2x + 5\)

24. \(x + 7y = 45\)

25. In class

26. In class
## 3.1 Graphing

### “Pick & Stick”
1. Pick any number for ‘x’ (or ‘y’)
2. Stick it in
3. Solve for ‘y’ (or ‘x’)
4. Graph point
5. Repeat

### Intercepts
An x-intercept happens when y is zero, and a y-intercept happens when x is zero.

### Vertical and Horizontal Lines
All equations that only have an ‘x’ will be vertical.
All equations that only have an ‘y’ will be horizontal.

## 3.2 Slope

### Formulas for Slope

\[
m = \frac{y_1 - y_2}{x_1 - x_2} \quad m = \frac{y \text{ change}}{x \text{ change}} \quad m = \frac{\text{rise}}{\text{run}} \quad m = \frac{\Delta y}{\Delta x}
\]

### Vertical Line
\[m = \text{undefined}\]

### Horizontal Line
\[m = 0\]

## 3.3 Graphing with Slope

### Standard Form
\[Ax \pm By = C\]

### Formula for Slope
\[m = -\frac{A}{B}\]

### Slope-Intercept Form
\[y = mx + b\]

### Slope
\[m = \text{coefficient of } x\]
3.4 Writing Equations

**Given Two Points**
1. Use the points to find the slope
2. If asked to write in standard form:
   a. Remember that \( m = -\frac{A}{B} \). Use the slope to fill in A & B in \( Ax + By = C \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for C
   d. Re-write with only \( x \) and \( y \) as variables
3. If asked to write in slope-intercept form:
   a. Fill in the slope for \( m \) in \( y = mx + b \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for \( b \)
   d. Re-write with only \( x \) and \( y \) as variables

**Given Slope and One Point**
Follow the same process as if you were given two points, but now you don’t have to find the slope.

3.5 Parallel and Perpendicular Slopes

| PARALLEL LINES HAVE THE SAME SLOPE | PERPENDICULAR LINES HAVE NEGATIVE RECIPROCALS FOR THEIR SLOPE |

**Given a Perpendicular or Parallel Line and a Point**
*This process is the same as solving given two points.*
4. Use the line to find the slope (parallel lines: same slope, perpendicular lines: slope is negative reciprocal)
5. If asked to write in standard form:
   a. Remember that \( m = -\frac{A}{B} \). Use the slope to fill in A & B in \( Ax + By = C \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for C
   d. Re-write with only \( x \) and \( y \) as variables
6. If asked to write in slope-intercept form:
   a. Fill in the slope for \( m \) in \( y = mx + b \)
   b. Fill in \( x \) and \( y \) using one point
   c. Solve for \( b \)
   d. Re-write with only \( x \) and \( y \) as variables
1. Create a visual chart of all of the methods, formulas, and examples from studying how to solve these linear equations.

Fill out the table for each of the following.

2. \(2x + 3y = 4\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>

3. \(9x - 5y = -160\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>71.78</td>
</tr>
<tr>
<td></td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>212</td>
</tr>
</tbody>
</table>

Give three points along each of the given lines.

4. \(y = 0.25x - 4\)

5. \(7x - 2y = -3\)

6. \(x - y = -2\)

Find the slopes of the given points.

7. \((\frac{1}{3}, \frac{4}{5})\) \((\frac{2}{3}, \frac{7}{5})\)

8. \((-3, \frac{2}{3})\) \((0, \frac{2}{3})\)

9. \((2, -3)\) \((-6, -7)\)

Convert to standard form if in slope-intercept form. Convert to slope-intercept form if in standard form.

10. \(y = \frac{4}{3}x + 1\)

11. \(y = \frac{-7}{6}x - 2\)

12. \(4x - 3y = -2\)

13. \(5x - 9y = -18\)

14. \(y = 3x + 1\)

15. \(y = \frac{-9}{10}x - 1\)

Write a line given the following information.

16. \((2, -4)\) \((8, 8)\)

17. \((2, 8)\) \((4, -2)\)

18. \(m = \frac{5}{6}\) \((1, \frac{13}{3})\)

19. \(m = -2\) \((-\frac{4}{3}, 2)\)

Write an equation that passes through the given point and is perpendicular to the given line equation.

20. \((7, 8)\) \(3x - 2y = 3\)

21. \((-1, -3)\) \(4x + 2y = -7\)

22. \((4, 2)\) \(y = 2\)

23. \((5, -1)\) \(y = \frac{5}{4}x + 2\)
Tell whether the lines are parallel, perpendicular or neither.

24. \(2x + 3y = 18\)  
   \(3x - 2y = 8\)

25. \(7x - 6y = 4\)  
   \(7x - 6y = -12\)

26. \(3x - 4y = 4\)  
   \(4x - 3y = -9\)

27. \(2x + 16y = 3\)  
   \(y = 8x - 2\)

28. \(3x - y = -4\)  
   \(y = -\frac{1}{3}x - 6\)

29. \(y = 5x + 4\)  
   \(y = \frac{1}{5}x + 3\)

Give the equation for the line that goes through the given point and is either parallel or perpendicular to the given line. Graph them together.

30. \(\quad y = \frac{1}{3}x + 2\)
    
    Parallel (3,8)

31. \(\quad y = -\frac{1}{5}x - 3\)
    
    Perpendicular (−1,−4)

32. \(\quad 4x - y = -2\)
    
    Parallel (−6,3)

33. \(\quad x - 3y = -3\)
    
    Perpendicular (2,−2)

34. \(\quad y = -2x + \frac{1}{2}\)
    
    Parallel (−3,3)

    Perpendicular (−2,−1)

35. \(\quad x + y = -2\)
    
    Parallel \(\left(\frac{3}{2}, \frac{5}{2}\right)\)

    Perpendicular (4,0)
Answers
1. Make it good.

2. | x  | y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>-3</td>
<td>10/3</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

3. | x  | y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>-40</td>
</tr>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>22.1</td>
<td>71.78</td>
</tr>
<tr>
<td>90</td>
<td>194</td>
</tr>
<tr>
<td>100</td>
<td>212</td>
</tr>
</tbody>
</table>

4. \((-1, -4.25), (0, -4), (4, -3)\)
5. \((-1, -2), \left(0, \frac{3}{2}\right), (1,5)\)
6. \((-1,1), (0,2), (1,3)\)
7. \(m = \frac{9}{5}\)
8. \(m = 0\)
9. \(m = \frac{1}{2}\)
10. \(4x - 3y = -1\)
11. \(7x + 6y = -12\)
12. \(y = \frac{4}{3}x + \frac{2}{3}\)
13. \(y = \frac{5}{9}x + 2\)
14. \(3x - y = -1\)
15. \(9x + 10y = -10\)
16. \(y = 2x - 8\)
17. \(y = -5x + 18\)
18. \(y = \frac{5}{6}x + \frac{7}{2}\)
19. \(y = -2x - \frac{2}{3}\)
20. \(y = -\frac{2}{3}x + \frac{38}{3}\)
21. \(y = \frac{1}{2}x - \frac{5}{2}\)
22. Undefined
23. \(y = -\frac{4}{5}x + 3\)
24. Perpendicular
25. Parallel
26. Neither
27. Perpendicular
28. Perpendicular
29. Neither

30. \(y = \frac{1}{3}x + 7\)
31. \(y = 5x + 1\)
32. \(y = 4x + 27\)
33. \(y = -3x + 4\)
34. Par: \(y = -2x - 3\) Perp: \(y = \frac{1}{2}x\)
35. Par: \(y = -x + 4\) Perp: \(y = x - 4\)
CHAPTER 3 REVIEW 2

Fill out the table for each of the following.

1. \( y = -\frac{2}{3}x + 1 \)  
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & \phantom{0} \\
   -2 & \phantom{0} \\
   1 & \phantom{0} \\
   -\frac{1}{3} & \phantom{0} \\
   -\frac{2}{3} & \phantom{0} \\
   \end{array}
   \]

2. \( 4x - 5y = 5 \)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -\frac{2}{5} & \phantom{0} \\
   -2 & \phantom{0} \\
   0 & \phantom{0} \\
   \frac{1}{3} & \phantom{0} \\
   \frac{7}{3} & \phantom{0} \\
   \end{array}
   \]

Give three points along each of the given lines

3. \( y = -\frac{3}{4}x + 2 \)

4. \( 7x + 6y = 9 \)

5. \( 8x - 6y = -1 \)

Find the slopes of the given points

6. \((-3,2)\) \((1,1)\)

7. \((\frac{6}{7}, 2)\) \((\frac{34}{7}, -5)\)

8. \((-1,-2)\) \((-3,-5)\)

Convert to standard form if in slope-intercept form. Convert to slope-intercept form if in standard form.

9. \( y = -\frac{1}{3}x + 2 \)

10. \( y = \frac{1}{4}x + \frac{5}{6} \)

11. \( 2x + 3y = -1 \)

12. \( 2x + 6y = 8 \)

13. \( 4x - y = 2 \)

14. \( y = -\frac{5}{4}x + 2 \)

Write a line given the following information

15. \((-\frac{4}{3}, \frac{1}{3})\) \((-\frac{1}{2}, \frac{5}{6})\)

16. \((\frac{7}{8}, 1)\) \((\frac{13}{8}, -\frac{9}{2})\)

17. \( m = -2 \) \((\frac{3}{2}, \frac{7}{2})\)

18. \( m = \frac{1}{2} \) \((2,4)\)

Write an equation that passes through the given point and is perpendicular to the given line equation.

19. \((0, -6)\) \( y = -4x - 3 \)

20. \((0,0)\) \( 4x + 7y = 0 \)

21. \((-1,1)\) \( y = 2x \)

22. \((3, -2)\) \( y = 6x - 5 \)
Tell whether the lines are parallel, perpendicular or neither

23. \(2x + 3y = 4\)  
   \(3x + 2y = 1\)

24. \(2x + 3y = 4\)  
   \(3x - 2y = -3\)

25. \(2x + 3y = 4\)  
   \(2x + 3y = -3\)

26. \(2x + 3y = 4\)  
   \(9x - 6y = 4\)

27. \(4x + 6y = 12\)  
   \(y = -\frac{2}{3}x - 1\)

28. \(2x + 3y = 4\)  
   \(y = \frac{1}{5}x + 3\)

Give the equation for the line that goes through the given point and is either parallel or perpendicular to the given line. Graph them together.

29. \(y = -\frac{3}{5}x + 2\)  
   Parallel (-1,2)

30. \(y = \frac{2}{3}x + \frac{1}{6}\)  
   Perpendicular (2,4)

31. \(7x - 2y = -1\)  
   Parallel (-2,3)

32. \(7x - 3y = -2\)  
   Perpendicular \((\frac{7}{3}, 7)\)

33. \(y = 3x + \frac{5}{8}\)  
   Parallel (4,-5)  
   Perpendicular (1,-1)

34. \(9x - 6y = -4\)  
   Parallel (1,3)  
   Perpendicular (2,5)
Answers

1. \[
\begin{array}{c|c}
  x & y \\
  \hline
  -3 & 3 \\
  -2 & \frac{7}{3} \\
  0 & 1 \\
  2 & -\frac{1}{3} \\
  4 & -\frac{5}{3} \\
\end{array}
\]

2. \[
\begin{array}{c|c}
  x & y \\
  \hline
  -4 & -\frac{21}{5} \\
  -2 & \frac{13}{5} \\
  0 & -1 \\
  2 & \frac{3}{5} \\
  5 & 3 \\
\end{array}
\]

3. \((-4,5), (0,2), (4, -1)\)

4. \((-1, \frac{8}{3}), (0, \frac{3}{2}), (1, \frac{1}{3})\)

5. \((-1, -\frac{7}{6}), (0, \frac{1}{6}), (1, \frac{3}{2})\)

6. \(m = -\frac{1}{4}\)

7. \(m = -\frac{7}{4}\)

8. \(m = \frac{3}{2}\)

9. \(x + 3y = 6\)

10. \(6x - 24y = -20\)

11. \(y = -\frac{2}{3}x - \frac{1}{3}\)

12. \(y = -\frac{1}{3}x + \frac{4}{3}\)

13. \(y = 4x - 2\)

14. \(5x + 4y = 8\)

15. \(y = \frac{3}{5}x + \frac{17}{15}\)

16. \(y = \frac{14}{3}x - \frac{37}{12}\)

17. \(y = -2x + \frac{13}{2}\)

18. \(y = \frac{1}{2}x + 3\)

19. \(y = \frac{1}{4}x - 6\)

20. \(y = \frac{7}{4}x\)

21. \(y = -\frac{1}{2}x + \frac{1}{2}\)

22. \(y = -\frac{1}{6}x - \frac{3}{2}\)

23. Neither

24. Perpendicular

25. Parallel

26. Perpendicular

27. Parallel

28. Neither

29. \(y = -\frac{3}{5}x + \frac{7}{5}\)

30. \(y = -\frac{3}{2}x + 7\)

31. \(y = \frac{7}{2}x + 10\)

32. \(y = -\frac{3}{7}x + 8\)

33. Par: \(y = 3x - 17\) Perp: \(y = -\frac{1}{3}x - \frac{2}{3}\)

34. Par: \(y = \frac{3}{2}x + \frac{3}{2}\) Perp: \(y = -\frac{2}{3}x + \frac{19}{3}\)
Chapter 4: EXPONENTS & POLYNOMIALS

Overview

Exponents and Scientific Notation
4.1 Laws of Exponents, Scientific Notation

Polynomials
4.2 Intro to Polynomials; Add/Subt. Polynomials
4.3 Multiply Polynomials
4.4 Division of Polynomials
“Soon after high school I married and a year and a half later I had my first baby. I loved being a mother and was perfectly content to stay at home with my children. However, after 9 ½ years I discovered that my marriage was not at all what I had believed it to be and I became a single mother of 4 small children ages 8 and younger. I found myself in the position of needing to support myself and my children financially. Not surprisingly, with only a high school education I could not find a job that would allow me to support myself and my children, so I decided to return to school 12 years after high school graduation. I was more scared of returning to school than I was of getting a divorce (and that scared me more than anything had in my entire life up to that point), and the idea of doing “college math” was the scariest part of college in my mind.”

“For as long as I remember I have struggled to understand and learn math concepts. I remember very clearly in the 5th grade my teacher, Mr. Clark, telling me in front of the entire class that I was stupid and I would never amount to anything. I was totally humiliated, and I’m sure I cried myself to sleep that night. A few years ago when I was telling my Mom what had happened she asked me why I didn’t tell her then and I realized it was because I believed him, and I didn’t want my Mom and Dad to know that I was stupid. I never did learn my times tables and in high school I didn’t get past pre-algebra and from the 5th grade on I never received higher than a C in a math class.”

“So now returning to college as a single mother, I started out in the 2nd to lowest math class available to me, Beginning Algebra. The only thing lower was Basic College Mathematics, but that class was full. I remember the first day of class. The professor passed out a pre-test. I looked at the test and wanted to cry, there were 20-30 questions and out of all of them I think I figured out 1 answer. I sat there thinking, ‘this is exactly what I thought college would be like’ and I was certain that I would fail.”

“Well to make what could be a very long story shorter I struggled and worried but kept trying all through the semester. I did everything the professor told us to do including completing each and every assignment on time, showing all my work on each problem and not trying to take short cuts, taking the opportunity to spend a great deal of time in the math lab and utilizing the help from the tutors there, making a visual chart for each chapter, and never missing class. I was pleased with my grade at the end of the semester. For the first time in my life I received an A in a math class. It was an awesome feeling. For the first time I had tasted success in school and I liked it and nothing else would ever be good enough again. The foundation for learning that I received in my pre-college algebra class completely changed my college career and I can honestly say that it changed my entire outlook on life. Doing well in school changed how I felt about myself as a person. I no longer accept or assume that I am stupid or incapable of learning. I still struggle and I have to work very hard for the grades that I receive but mine is a success story. Although I am not yet finished with my education I was recently accepted into the nursing program at my university with a 3.9 GPA.”

Karin Rice, 2007
4.1 Laws of Exponents, Scientific Notation

OBJECTIVES
• Learn and apply laws of exponents, including the zero and one rule, the product rule, the quotient rule, the negative exponent rule, and the power rule

DEFINITIONS & BASICS

EXPONENT/POWER: The shorthand to explain how many times something is multiplied by itself (remember, the number being multiplied by itself is called the base.)

Example: \(2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128\) is the same as \(2^7 = 128\)

ROOT: Like working an exponent backwards, roots find out what number or variable was raised to a power.

Example: We know that \(2^7 = 128\), so to work it backwards, we say:

\[2 = \sqrt[7]{128}\]

or if 128 was cut into 7 equal parts multiplied together, then those parts would all be 2’s.

LAWS & PROCESSES

Sometimes we have exponential expressions that can be simplified. We have five rules that help us simplify these exponential expressions that we will call the LAWS OF EXPONENTS. Again, these rules or “laws” are shortcuts. We will learn the first four in this section.

**Simplifying Exponential Expressions**

Apply the LAWS of EXPONENTS as outlined below until:
1. All exponents of zero and one are gone
2. All products and quotients with the same base are united
3. No negative exponents are left
4. All powers being raised to powers are multiplied

There’s no order in which we use these rules. We simply apply them as needed.
<table>
<thead>
<tr>
<th>LAW</th>
<th>HYPOTHETICAL</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ZERO AND ONE RULE</strong></td>
<td><em>a^0 = 1</em> and <em>a^1 = a</em></td>
<td><em>7^0 = 1</em> and <em>7^1 = 7</em></td>
</tr>
<tr>
<td>Anything to the power of zero = 1 (if a ≠ 0) and Anything to the power of one = itself</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>PRODUCT RULE</strong></td>
<td><em>a^m \cdot a^n = a^{m+n}</em></td>
<td><em>x^2 \cdot x^3 = x^{2+3}</em></td>
</tr>
<tr>
<td>Exponents being multiplied with the same base are added.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| **QUOTIENT RULE**          | *\frac{a^m}{a^n} = a^{m-n} \text{ where } m > n* | *\frac{x^5}{x^3} = x^{5-3} = x^2*  
OR
*\frac{x^2}{x^5} = \frac{1}{x^{5-2}} = \frac{1}{x^3}*  
AND
*\frac{x^3}{x^3} = x^{3-3} = x^0 = 1*
| Exponents being divided with the same base are subtracted. (if a ≠ 0) |                                |                          |
| **NEGATIVE EXPONENT RULE** | *a^{-m} = \frac{1}{a^m}*     | *x^{-3} = \frac{1}{x^3}*
AND
*\frac{1}{a^{-m}} = a^m*  
AND
*\frac{1}{x^{-3}} = x^3*     | All negative exponents can be converted to a positive exponent by simply taking the reciprocal. (if a ≠ 0)
This must be done to all negative exponents in order to consider a problem simplified, with the exception of scientific notation (see section 4.2) |                          |
<p>| <strong>POWER RULE</strong>             | <em>(a^m)^n = a^{mn}</em>            | <em>(x^2)^3 = x^{2\cdot3} = x^6</em> |
| To raise a power to a power, we multiply the exponents. |                                |                          |
| IMPORTANT: Every base in the parentheses ( ) receives the power. |                                |                          |</p>
<table>
<thead>
<tr>
<th>LAW</th>
<th>WHY? (HOW IS IT A SHORTCUT?)</th>
<th>COMMON ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero and One</td>
<td>(See explanation in quotient rule)</td>
<td>• $7^0 \neq 0$; A tendency to want to multiply by zero when seeing a degree of zero</td>
</tr>
</tbody>
</table>
| Product     | Since $x^2 = x \cdot x$ and $x^3 = x \cdot x \cdot x$  
Then we know…. $x^2 \cdot x^3 = x^5$  
$x \cdot x \cdot x \cdot x \cdot x = x^5$ | • $x^2 \cdot x^3 \neq x^6$  
A tendency to want to multiply, not add, the exponents  
$x^2 \cdot y^3 = x^2 y^3 \neq (xy)^5$  
Product and quotient rules apply only to exponents w/same base. |
| Quotient    | $\frac{x^5}{x^3} = \frac{\cancel{x} \cdot x \cdot \cancel{x} \cdot x}{\cancel{x}} = \frac{x^2}{x^0} = x^2$  
OR  
$x^2 = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{x^0}{x^3} = \frac{1}{x^3}$  
AND  
$x^3 = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$ or $x^0 = 1$ | • $x^9 \neq x^3$  
A tendency to want to divide when exponents work as simple whole # quotients |
| Negative Exponent | In the quotient rule we learn:  
$\frac{x^2}{x^5} = \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3} = \frac{1}{x^{5-2}}$  
What if we don’t subtract larger-smaller exponent:  
We get $\frac{x^2}{x^5} = x^{2-5} = x^{-3}$ instead of:  
$\frac{1}{x^{5-2}} = \frac{1}{x^3}$  
Hence, $\frac{1}{x^3} = x^{-3}$ | • $-2x^{-3} \neq 1 \frac{1}{2x^3}$  
• $-2x^{-3} = \frac{-2}{x^3}$  
The negative exponent rule states we can’t have negative exponents in a simplified expression. However, negative coefficients have always been acceptable and are not reciprocated and made positive. Hence the -2 stays negative and stays on top. |
| Power       | $x^2 = x \cdot x$  
$(x^2)^3 = (x^2)(x^2)(x^2) = x^6$  
$x \cdot x \cdot x \cdot x \cdot x \cdot x$ | • $(2x)^4 \neq 2x^4$  
vs.  
$(2x)^4 = 2^4 x^4 = 16x^4$  
Remember, every base in the parentheses receives the power. |
# Section 4.1

Using the Laws of Exponents, simplify the following:

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>SOLUTION</th>
<th>LAW(S) USED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^1$</td>
<td>5</td>
<td>One Rule</td>
</tr>
<tr>
<td>$x^1$</td>
<td>x</td>
<td>One Rule</td>
</tr>
<tr>
<td>$(2x)^0$</td>
<td>1</td>
<td>Zero Rule</td>
</tr>
<tr>
<td>$8^3 \cdot 8^{19}$</td>
<td>$8^{3+19} = 8^{22}$</td>
<td>Product Rule</td>
</tr>
<tr>
<td>$x^2 \cdot x^7$</td>
<td>$x^{2+7} = x^9$</td>
<td>Product Rule</td>
</tr>
<tr>
<td>$4^{-2}$</td>
<td>$\frac{1}{4^2} = \frac{1}{16}$</td>
<td>Negative Exponent Rule</td>
</tr>
<tr>
<td>$d^{-3} \cdot d^8$</td>
<td>$d^{8-3} = d^5$</td>
<td>Product Rule</td>
</tr>
<tr>
<td>$\frac{x^{16}}{x^{16}}$</td>
<td>$x^{16-16} = x^0 = 1$</td>
<td>Quotient &amp; Zero Rules</td>
</tr>
<tr>
<td>$\frac{b^7}{b^5}$</td>
<td>$b^{7-5} = b^2$</td>
<td>Quotient Rule</td>
</tr>
<tr>
<td>$\frac{p^{-2}}{p^{-5}}$</td>
<td>$p^{-2-(-5)} = p^3$</td>
<td>Quotient Rule</td>
</tr>
<tr>
<td>$\frac{x^2}{x^3}$</td>
<td>$\frac{1}{x^{3-2}} = \frac{1}{x^1} = \frac{1}{x}$</td>
<td>Quotient &amp; One Rule</td>
</tr>
<tr>
<td>$(3x^3)^2$</td>
<td>$3^{1\cdot2}x^{3\cdot2} = 3^2x^6 = 9x^6$</td>
<td>Power Rule</td>
</tr>
<tr>
<td>$(2x^2y^3z^4)^3$</td>
<td>$2^{1\cdot3}x^{2\cdot3}y^{3\cdot3}z^{4\cdot3} = 2^3x^6y^9z^{12} = 8x^6y^9z^{12}$</td>
<td>Power Rule</td>
</tr>
<tr>
<td>$(5x^2y^{-3})^2$</td>
<td>$5^{1\cdot2}x^{2\cdot2}y^{-3\cdot2} = 5^2x^4y^{-6} = \frac{25x^4}{y^6}$</td>
<td>Power Rule &amp; Negative Exponent Rule</td>
</tr>
</tbody>
</table>
Remember…

**Simplifying Exponential Expressions**

As you do these problems ask yourself…

1. Do I have any exponents of zero and one?
2. Do I have any products and quotients with the same base?
3. Are there any negative exponents?
4. Are there any powers being raised to powers?

Once you can answer no to all of these, your exponential expression problem is finished.

**One more thing…**

Simplify the following expressions:

\[
(-1)^2 = 1 \\
(-1)^3 = -1 \\
(-1)^4 = 1 \\
(-1)^98 = 1 \\
(-1)^99 = -1
\]

This is an interesting pattern:

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Expanded</th>
<th>Evaluated</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1)^1)</td>
<td>((-1))</td>
<td>(-1)</td>
</tr>
<tr>
<td>((-1)^2)</td>
<td>((-1)(-1))</td>
<td>(1)</td>
</tr>
<tr>
<td>((-1)^3)</td>
<td>((-1)(-1)(-1))</td>
<td>(-1)</td>
</tr>
<tr>
<td>((-1)^4)</td>
<td>((-1)(-1)(-1)(-1))</td>
<td>(1)</td>
</tr>
</tbody>
</table>

The pattern develops a **RULE:**

**Rule for Negative Bases**

When a base is negative…

- an even exponent yields a **positive** answer
- an odd exponent yields a **negative** answer

**EXAMPLES**

\[
(-x)^{30} = (1-x)^{30} = -1^{1\cdot30}x^{1\cdot30} = -1^{30}x^{30} = x^{30}
\]

Again, in this problem, 1) we see that the power goes to all the bases. 2) We have to recognize that the negative \((-\) is also a base and can be expressed as \(-1, and 3) we recognize again that when a base does not have a power written next to it, we can assume that power is 1.
In examples 4 & 5, note that the every number in the parentheses receive the power, including the bases in a quotient, the whole numbers and the variables, and the top and bottom of the fraction.

4

\[
\left( \frac{2x^4}{y^4} \right)^3 = \frac{2^{1 \cdot 3} x^{4 \cdot 3}}{y^{4 \cdot 3}} = \frac{2^3 x^{12}}{y^{12}} = \frac{8x^{12}}{y^{12}}
\]

5

\[
\left( \frac{p^2}{4} \right)^{-3} = \frac{p^{2(-3)}}{4^{1(-3)}} = \frac{p^{-6}}{4^{-3}} = \frac{4^3}{p^6} = \frac{64}{p^6}
\]

There is an alternative way to do this problem…

\[
\left( \frac{p^2}{4} \right)^{-3} = \left( \frac{4}{p^2} \right)^3 = \frac{4^{1 \cdot 3}}{p^{2 \cdot 3}} = \frac{4^3}{p^6} = \frac{64}{p^6}
\]

C

**SCIENTIFIC NOTATION:** To save us the hassle of writing too many 0’s before or after a number that is really big or really small, then we just multiply the main part of the number by 10^a, and “a” can be any number that you need it to be.

**Example:**

\[53,400,000,000 = 5.34 \times 10^{10}\]

Notice how all those decimal places just get sucked up into the exponent. It is *official scientific notation when the first number is between 1 and 9.*

**I. Multiplying and Dividing in Scientific Notation**

**Steps of Multiplying and Dividing Sci. Notation**

1. Combine number with numbers (× or ÷)
2. Combine base 10 with base 10 (× or ÷ using laws of exponents)
3. Write answer in scientific notation if it’s not already
Evaluate \((4.8 \times 10^4)(3.7 \times 10^{-15})\)

\[
4.8 \times 3.7 = 17.76 \\
10^4 \times 10^{-15} = 10^{4+(-15)} = 10^{-11} \\
17.76 \times 10^{-11} \\
17.76 = 1.776 \times 10^1 \\
17.76 \times 10^{-11} = 1.776 \times 10^1 \times 10^{-11} \\
= 1.776 \times 10^{-10}
\]

Step 1: Combine numbers with numbers
Step 2: Combine base 10 with base 10
Put the first two steps together
Step 3: Write answer in scientific notation if needed

**COMMON ERRORS**

1. Don’t convert 1.776 \(\times\) 10\(^{-10}\) to \(\frac{1.776}{10^{10}}\). Remember in Scientific Notation it’s ok to have negative exponents.
2. When converting 17.76 to scientific notation, be careful. It is easy to make an error on the new exponent of base 10 (i.e in the above example we could easily get 10\(^{-12}\) instead of getting 10\(^{-10}\)…can you see how?)

Evaluate \(\frac{2.5 \times 10^2}{12.5 \times 10^5}\)

\[
\frac{2.5}{12.5} = 0.2 \\
\frac{10^2}{10^5} = 10^{2-5} = 10^{-3} \\
= 0.2 \times 10^{-3} \\
0.2 = 2.0 \times 10^{-1} \\
0.2 \times 10^{-3} = 2.0 \times 10^{-1} \times 10^{-3} \\
= 2.0 \times 10^{-4}
\]

Step 1: Combine numbers with numbers
Step 2: Combine base 10 with base 10
Put the first two steps together
Step 3: Write answer in scientific notation if needed
Find the equation of the line that is parallel or perpendicular to the given line and passes through the point (0,2).

1. Parallel to $3x + 4y = 12$
2. Perpendicular to $y = -\frac{5}{3}x - 3$

Use positive exponents to express the following.

3. $\frac{1}{k^{-9}}$
4. $a^5b^{-3}c^{-2}$
5. $\frac{z^{-1}}{x^{-3}y^{-12}}$

Use negative exponents to express the following.

6. $\frac{1}{x^2}$
7. $\frac{3}{b^5}$
8. $\frac{1}{4^2}$

Perform the indicated operation and simplify.

9. $x^6 \cdot x^3$
10. $2^3 \cdot 2^4$
11. $\frac{r^3}{r^2}$
12. $ab^0$
13. $y^{-7} \cdot y^5$
14. $\frac{1}{m^{-5}} \cdot m^8$
15. $(8k - 7k)^3 \cdot k^{-9}$
16. $\frac{x^7y^{-3}}{x^{-4}z_{12}}$
17. $(t)^4 \cdot (t)^0$
18. $(-1)^{53} \cdot (-1)^2$
19. $(a^5b^3) \cdot (a^2b^{-7})$
20. $\frac{x^{-7}}{x^4}$
21. $\frac{h^7f^{-3}}{h^{-4}f^{12}}$
22. $\frac{Q^{-8}}{Q^{-12}}$
23. $(-1)^7 \cdot (\frac{m^4}{m^2})$
24. $\frac{g^5j^{-8}o^{-9}r^{19}t^8e^1}{b^2e^{-12}a^{-21}r^3}$
25. $(gf)^7$
26. $(8d)^3$
27. $(6x)^{-5}$
28. $(4x^{-2}y^{-9})^3$
29. $\left(\frac{2}{c^8}\right)^{-3}$
30. $\left(\frac{m^4}{m^{-3}}\right)^2$
31. $\left(\frac{9k^{-5}j^3}{l^{12}}\right)^{-4}$
32. $\left(\frac{3j^5}{7j^8h^{-1}}\right)^3$

Multiply. Round to four decimal places.

33. $4.2 \times 10^{12} \cdot 1.8 \times 10^{-4}$
34. $3.02 \times 10^8 \cdot 7.301 \times 10^{-5}$

Divide. Round to four decimal places.

35. $\frac{8 \times 10^7}{4 \times 10^5}$
36. $\frac{3.75 \times 10^6}{2.5 \times 10^9}$
37. $\frac{5.759 \times 10^{-7}}{3.51 \times 10^4}$
38. $\frac{9.981 \times 10^{13}}{4.34 \times 10^{-19}}$

Preparation

39. Classify each of the following as a monomial, binomial, trinomial, or polynomial.
   a) $x + 3$
   b) $x + y + z$
   c) 5
Answers:

1. \( y = -\frac{3}{4}x + 2 \)
2. \( y = \frac{3}{5}x + 2 \)
3. \( k^9 \)
4. \( \frac{a^5}{b^3c^2} \)
5. \( \frac{x^3y^{12}}{z} \)
6. \( x^{-2} \)
7. \( 3B^{-5} \)
8. \( 4^{-2} \)
9. \( x^9 \)
10. 128
11. \( r \)
12. \( a \)
13. \( \frac{1}{y^2} \)
14. \( m^{13} \)
15. \( \frac{1}{k^6} \)
16. \( \frac{x^3}{y^3z^{12}} \)
17. \( t^4 \)
18. -1
19. \( \frac{a^7}{b^4} \)
20. \( \frac{1}{x^{14}} \)
21. \( \frac{h^{11}}{f^{15}} \)
22. \( Q^4 \)
23. \( -m^2 \)
24. \( \frac{g^5r^{16}e^{13}a^{21}k^8}{j^{8o^9b^2}} \)
25. \( g^7f^7 \)
26. 512d^3
27. \( \frac{1}{7776x^5} \)
28. \( \frac{64}{x^6y^{27}} \)
29. \( \frac{c^{24}}{8} \)
30. \( m^8n^6 \)
31. \( \frac{k^{20}l^{48}}{6561j^{12}} \)
32. \( \frac{27h^3}{343f^9} \)
33. \( 7.56 \times 10^8 \)
34. \( 2.2049 \times 10^4 \)
35. \( 2 \times 10^2 \)
36. \( 1.5 \times 10^{-3} \)
37. \( 1.6407 \times 10^{-11} \)
38. \( 2.2998 \times 10^{32} \)
39. In class.

Section 4.1
4.2 Intro to Polynomials; Add/Subtract Polynomials

**OBJECTIVES**
- Recognize polynomials and the different types
- Recognize terms, coefficients, and degrees
- Simplify polynomials by combining like terms

### A POLYNOMIALS AND TERMS

#### DEFINITIONS & BASICS

**POLYNOMIAL**: “Poly” means *many*, and “nomial” means *term*, so a polynomial is something with many terms.

**TERM**: A term is simply a quantity. It can be expressed as any of the following…

- $7x$
- $7xy$
- $7x/y^2$

<table>
<thead>
<tr>
<th>A Number</th>
<th>A variable</th>
<th>A number and a variable</th>
<th>A number and multiple variables</th>
<th>An exponential expression</th>
<th>A fraction</th>
</tr>
</thead>
</table>

**Principles About Terms**

1. Terms are separated by plus (+) and minus (-) signs.
   a. Hence $7x - 5y$ has two terms, and $x^2 + 5x - 6$ has three terms

2. The sign before the term always goes with it.
   a. Hence the two terms in $7x - 5y$ are $7x$ and $-5$, and the three terms in $x^2 + 5x - 6$ are $x^2$, $5x$, and $-6$.

The family of polynomials includes the *monomials*, *binomials*, and *trinomials*. Monomials, binomials, and trinomials are all *polynomials* with either one, two, or three terms respectively:

<table>
<thead>
<tr>
<th>MONOMIAL</th>
<th>BINOMIAL</th>
<th>TRINOMIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mono = One Nomial = Term(s) “One Term”</td>
<td>Bi = Two Nomial = Term(s) “Two Terms”</td>
<td>Tri = Three Nomial = Term(s) “Three Terms”</td>
</tr>
</tbody>
</table>

So… are there “quadnomials, heptnomials, sexnomials, septnomials, octnomials”, indicating polynomials with 4,5,6,7,8 terms? No: Anything with 4 or more terms we just call a *polynomial*.

Section 4.2
## MONOMIAL
- **(One Term)**
  - 7
  - $x$
  - $-7x$
  - $\frac{7x}{y^2}$

## BINOMIAL
- **(Two Terms)**
  - $7 + y$
  - $a - b$
  - $14 - 2ab$
  - $\frac{1}{2} + 4a^2b$

## TRINOMIAL
- **(Three Terms)**
  - $7 - 2ab + b$
  - $x + y + 1$
  - $2a + b - 4c$
  - $2x^2y - 4xy + \frac{4}{5}$

## POLYNOMIAL
- **(Four or more terms)**
  - $x + 2x^2y - 2y + 17$
  - $2a + b - 4c + 14$
  - $2x^2 + x + 2y - z + 3$
  - $2x^2y - 4xy + \frac{1}{2}x - 7$

### ANATOMY OF A POLYNOMIAL: Coefficients & Degrees of Terms, Degree of Polynomial

#### COEFFICIENT:
This is the number to the left of the variable in every term. Every term has a coefficient. If no number is written we assume it’s a 1 as with the second term below.

**Example:** In the polynomial: $5x^2 + x - 5$

- The coefficient of the 1st term = 5, the 2nd term = 1, and the 3rd term = -5

**Remember, the sign before the term goes with it in indicating the sign of each coefficient.**

#### DEGREE:
This is the small number to the upper right of the variable in every term. If no degree is indicated next to a variable we assume it’s a 1 as with the second term below.

**Example:** In the same polynomial: $5x^2 + x - 5$

- The degree of the 1st term = 2, the 2nd term = 1, and the 3rd term = 0

**The degree of a term with no variable like the 3rd term above ($-5$) is always zero. When we speak of the degree of a term we are speaking of the degree of its variable(s) not the degree of its coefficients. Since there is no variable with -5, it can be defined as $-5x^0$**

**When a term has more than one variable, the sum of their degrees is the degree of the term i.e. the degree of $2xy^2$ is 3**

#### DEGREE OF POLYNOMIAL:
The greatest degree of any one term in the polynomial. Hence, the DEGREE OF POLYNOMIAL $5x^2 + x - 5$ is 2.
Find the following for this polynomial: $4x^5 - 12x + 9xy^2 - 7xy + y^2 - 10$

Coefficient of Each Term: 4, -12, 9, -7, 1, -10
Degree of Each Term: 5, 1, 3, 2, 2, 0
Degree of Polynomial: 5

C POLYNOMIAL ETIQUETTE: Descending Order

DEFINITIONS & BASICS

Principles of Descending Order

1. It is common practice to write all answers to polynomial problems in **descending order**.
2. **Descending order** is defined as largest to smallest
3. It is the **degrees** of the terms that we are ordering from largest to smallest.

EXAMPLES

2 Write in descending order: $2x^3 - 12x + 9x^4 - 7$
Answer: $9x^4 + 2x^3 - 12x - 7$

3 Write in descending order: $4x^5 - 3x - 10 + 9xy^2 - 7xy + 2x^7$
Answer: $2x^7 + 4x^5 + 9xy^2 - 7xy - 3x - 10$
EVALUATING POLYNOMIALS

Evaluating a polynomial is simply substitution & simplification – putting one value in for another & applying the order of operations.

**Example:** Evaluate the polynomial \(2x^2 - 3x + 5\), when \(x = -1\)

Everywhere there’s an \(x\) you’ll substitute in a -1…

\[
x^2 - 3x + 5
\]
\[
2(-1)^2 - 3(-1) + 5
\]

You’ll now simplify by applying the order of operations:

\[
2(-1)^2 - 3(-1) + 5
\]
\[
2(1) - 3(-1) + 5
\]
\[
2 + 3 + 5
\]
\[
10
\]

EXAMPLES

**Evaluate the polynomial** \(x^2 - 2xy + 3y^2\), when \(x = 2\) and \(y = -1\)

\[
x^2 - 2xy + 3y^2
\]
\[
(2)^2 - 2(2)(-1) + 3(-1)^2
\]
\[
4 - 2(-1) + 3(1)
\]
\[
4 + 4 + 3
\]
\[
11
\]

Answer: 11

POLYNOMIAL LIKE TERMS: Identify, Collect, Combine

We’ve worked with LIKE TERMS already in Ch.1 and Ch.2. Here’s a review.

**DEFINITIONS & BASICS**

**LIKE TERMS:** Terms with the same variable and degree combination. Take the following terms for example: \(2xy^2\), \(-2xy\), \(3y^2\). Are any like terms? No. While they are similar, the variables and degrees have to be exactly the same. The only thing that can differ is the coefficient.

**COMMON ERRORS:** Sometimes it is easy to mistakenly combine things that look a lot alike. For example, \(xy\) and \(xy^2\) and \(x^2y\) and \(x^2y^2\) are all different. They are not like terms.

**IDENTIFY LIKE TERMS:** When you have a series of terms, it is important to identify like terms so they can be collected and combined. It’s simple organizing. Using shapes and symbols like circles, squares, and underlines is helpful and keeps us from losing one along the way.
Section 4.2

EXAMPLES

5

Simplify: $5x^2 + 11x - 7 - 4x + 12 + 3x^2$

Identify: Using symbols, we organize $x^2$'s with □'s, $x$'s with ___'s, and numerical values with ○'s.

Collect: Write like terms next to each other. Now we can clearly see which terms combine.

Combine: How many $x^2$'s, $x$'s, and numbers do we have?

Answer: $8x^2 + 7x + 5$

6

Simplify: $2xy^2 - 5xy + 3y^2 + 7xy^2 + 8y^2 - 2xy$

Identify: Organize like terms using symbols.

Collect: Write like terms next to each other.

Combine: Add/Subtract like terms.

Answer: $9xy^2 - 7xy + 11y^2$

LAWS & PROCESSES

In Section 1.8 we learned how to simplify a problem by 1) distributing parenthesis and 2) combining like terms. Adding and subtracting POLYNOMIALS is the same process.

Adding and Subtracting Polynomials

1. Distribute parentheses. In addition, parentheses can just be dropped. In subtraction, the minus sign becomes a $(-1)$ distributor.
2. Combine like terms.

Section 4.2
Add: \((5x^4 - 3x^2 - 10) + (9x^4 + 4x^2 - 3)\)

Distribute Parentheses: Because it’s addition and there’s no distributor, we can just drop the ( ).

Identify: Organize like terms using symbols.

Collect: Write like terms next to each other.

Combine: Add/Subtract like terms.

Answer: \(14x^4 + x^2 + 7\)

Subtract: \((5x^4 - 3x^2 - 10) - (9x^4 + 4x^2 - 3)\).

Distribute Parentheses: The 2nd set of parenthesis has a distributor of \(-1\) which changes the sign of every term.

Identify: Organize like terms using symbols.

Collect: Write like terms next to each other.

Combine: Add/Subtract like terms.

Answer: \(4x^4 - 7x^2 + 13\)

A Negative One \((-1)\) Distributor

In the example above we had a \(-1\) distributor around the 2nd set of parentheses:

\[-(9x^4 + 4x^2 - 3)\]

This is the case in all subtraction of polynomial problems. The negative sign in front of the polynomial behaves like a \(-1\).

When we distribute that \(-1\), we get:

\[(-1)(9x^4) + (-1)(4x^2) + (-1)(-3)\]

which changes the sign of every term:

\[-9x^4 - 4x^2 + 3\]
4.2 EXERCISE SET

Simplify.

4.1

1. \((5a)^{-3}\)

2. \((2x^{-3}y^{-8})^6\)

3. \(\left(\frac{9k^{-5}k^{3}}{h^7}\right)^{-2}\)

4. Express in scientific notation: \(15,966,000,000,000\)

5. Express in decimal form: \(2.97 \times 10^{-9}\)

Perform the indicated operation.

6. \(5.3 \times 10^{-4} \cdot 1.01 \times 10^8\)

7. \(\frac{1.8 \times 10^{-8}}{6 \times 10^{-6}}\)

8. \(\frac{9.9 \times 10^{-7}}{3.3 \times 10^4}\)

Classify the following as monomials, binomials, trinomials, or polynomials.

4.2

9. \(x^2 - 10x + 25\)

10. \(18a^4b^3yz^9\)

11. \(-2a(b^6c) - xy\)

12. \(a^4 + a^2b^2 + ab^2 - b^3 + 5\)

Identify each term. Name the coefficient and power of each term, as well as the power of the polynomial.

13. \(9 + 3k\)

14. \(x^2 + 8x + 16\)

15. \(13x^3 + x^2 + 5x + 3\)

16. \(-2b^7 - 14b + 3\)

Write in descending order.

17. \(s + 7 + 3s^2\)

18. \(3x^2 + 5 + x^4 + 2x^3 + 4x\)

Evaluate.

19. \(x^2 - 10x + 25\)
   when \(x = 4\)

20. \(3a^4 + 4a^2 - 10a - 19\)
   when \(a = -5\)

21. \(a^4 + a^2b^2 + ab^2 - b^3 + 5\)
   when \(a = -1\) and \(b = 3\)

22. \(x^3 - 3x^2y + xy^2 + y\)
   when \(x = 2\) and \(y = -4\)

Simplify. Write your answer in descending order.

23. \(5x^2 + 3x + x - 9\)

24. \(4x - 6x^5 + 17x + 15x^5 + 3x - x^3\)

25. \(b^{19} - 4b^{14} + 5b^{20} - 2b^{14}\)

26. \((4k - 12k) + 5k^2 - 4\)

27. \(-x^8 - 5xy + 4xy^2 - 9x^3y + x^8\)

28. \(\frac{7}{2}y^2 + x^4 - \frac{3}{2}y^2 + \frac{1}{3}x^3 + 7y\)
Add the polynomials.

29. \((3x - 2) + (x + 5)\)  
30. \((4a) + (2a - 5)\)

31. \((-7x^2 + 5y - 17) + (3x^2 - 4x + 12y)\)  
32. \((5x^4 - x^3 + 3x^2 - 5) + (4x^4 + 4x^3 + x)\)

Subtract the polynomials.

33. \((5x + 2) - (4x + 3)\)  
34. \((3x^2 - x + 7) - (9x^2 + x + 8)\)

35. \((4y^7 + x^2 + 6y) - (6y^7 - 5y^5 + 11x^2 - y + 17)\)  
36. \((6a^3 - b^3 + b^2) - (-a^3 + 2b^3 - b^2)\)

37. \((5x^4 - x^3 + 3x^2 - 5) - (4x^4 + 4x^3 + x)\)  
38. \((x^2 + x + 1) - (-x^2 - 2x + 3)\)

Find a polynomial that describes the perimeter of these shapes.

39. \(4s^2 + s - 15\)

40. 

\[
\begin{align*}
\text{r} & \quad 3r + t \\
\text{t}^2 + r & \quad \text{t}^2 \\
\text{r} + 1 & \quad 3
\end{align*}
\]

Preparation:

41. Match the following three equations with the property that is being used.

\[
\begin{align*}
9(x - 2) &= 9x - 18 & \text{Commutative Addition} \\
3ab + 4t &= 4t + 3ab & \text{Associative Multiplication} \\
2(x5) &= (2x)5 & \text{Distributive}
\end{align*}
\]
Answers:

1. \( \frac{1}{125a^3} \)
2. \( \frac{64}{x^{18}y^{48}} \)
3. \( \frac{h^{14}k^4}{81} \)
4. \( 1.5966 \times 10^{13} \)
5. \( 0.00000000297 \)
6. \( 5.353 \times 10^4 \)
7. \( 3 \times 10^{-3} \)
8. \( 3 \times 10^{-11} \)
9. Trinomial
10. Monomial
11. Binomial
12. Polynomial
13. 9: coefficient = 9, power = 0
   3k: coefficient = 3, power = 1
   power of polynomial = 1
14. \( x^2 \): c = 1, p = 2
   8x: c = 8, p = 1
   16: c = 16, p = 0
   power of polynomial = 2
15. \( 13x^3 \): c = 13, p = 3
   \( x^2 \): c = 1, p = 2
   5x: c = 5, p = 1
   3: c = 3, p = 0
   power of polynomial = 3
16. \( -2b^7 \): c = -2, p = 7
   \( -14b \): c = -14, p = 1
   3: c = 3, p = 0
   power of polynomial = 7
17. \( 3s^2 + s + 7 \)
18. \( x^4 + 2x^3 + 3x^2 + 4x + 5 \)
19. 1
20. 2006
21. -21
22. 84
23. \( 5x^2 + 4x - 9 \)
24. \( 9x^5 - x^3 + 24x \)
25. \( 5b^{20} + b^{19} - 6b^{14} \)
26. \( 5k^2 - 8k - 4 \)
27. \( -9x^3y^3 + 4x^2y^2 - 5xy \)
28. \( x^4 + \frac{1}{3}x^3 + 2y^2 + 7y \)
29. \( 4x + 3 \)
30. \( 6a - 5 \)
31. \( -4x^2 - 4x + 17y - 17 \)
32. \( 9x^4 + 3x^3 + 3x^2 + x - 5 \)
33. \( x - 1 \)
34. \( -6x^2 - 2x - 1 \)
35. \( -2y^7 + 5y^5 - 10x^2 + 7y - 17 \)
36. \( 7a^3 - 3b^3 + 2b^2 \)
37. \( x^4 - 5x^3 + 3x^2 - x - 5 \)
38. \( 2x^2 + 3x - 2 \)
39. \( 8s^2 + 8s - 26 \)
40. \( 2t^2 + t + 6r + 4 \)
41. In class.

Section 4.2
4.3 Multiply Polynomials

OBJECTIVES

- Multiply monomials with monomials
- Multiply monomials with any polynomial
- Multiply binomials with binomials
- Multiply any polynomial with any polynomial
- Recognize special products of polynomials including sum & difference and square binomials

There are four types of multiplication of polynomials (see A-D). However, they all stem from learning the first type:

A TYPE I: MONOMIALS × MONOMIALS

LAWS & PROCESSES

Multiply MONOMIAL × MONOMIAL

1. Multiply numbers with numbers (coefficients)
2. Multiply like variables with like variables (x’s with x’s, y’s with y’s)

EXAMPLES

1 Multiply (4x)(5x)

\((4x)(5x)\) At a closer look, everything is being multiplied by everything else.
\(4 \cdot x \cdot 5 \cdot x\) Because of this, we can apply the commutative law and place numbers with numbers and like variables with like variables.
\(4 \cdot 5 \cdot x \cdot x\)
\(20x^2\)

Answer: \(20x^2\)

2 \((-4x^5)(6x^2)\)
\((-4)(6)x^5 \cdot x^2\)
\(-24x^{5+2}\)
\(-25x^7\)

3 \((4xy^2)(-2x^3y)\)
\((4)(-2)x \cdot x^3 \cdot y^2 \cdot y\)
\(-8x^{1+3}y^{2+1}\)
\(-8x^4y^3\)

4 \((-2y^2)(6y^3)(-4y^5)\)
\((-2)(6)(-4)y^2 \cdot y^3 \cdot y^5\)
\(48y^{2+3+5}\)
\(48y^{10}\)
From here on out in this section, any polynomial refers to a polynomial other than a monomial, like binomials and trinomials. The other three methods are now extensions of the monomial × monomial method.

**Multiply MONOMIAL × POLYNOMIAL**

3. Multiply the monomial (the distributor) times each term in the polynomial.
4. Format it such that you have: (mono)×(term) + (mono)×(term) + …
   keeping the sign of each term with it in the parenthesis

**EXAMPLES**

**5** Multiply: \(3x(5x - 4)\)

\[
3x(5x - 4) = 3x \text{ is the monomial distributor, being multiplied to the polynomial}
\]

\[
(3x)(5x) + (3x)(-4)
\]

\[
3 \cdot 5 \cdot x \cdot x + 3(-4) \cdot x
\]

\[
15x^2 - 12x
\]

**Answer:** \(15x^2 - 12x\)

**6** Multiply: \(-4x(-x - 4)\)

\[
-4x(-x - 4) = (-4x) \text{ is the distributor here, not just } (4x). \text{ The sign before the term goes with it.}
\]

\[
(-4x)(-x) + (-4x)(-4)
\]

\[
(-4)(-1) \cdot x \cdot x + (-4)(-4) \cdot x
\]

\[
4x^2 + 16x
\]

**Answer:** \(4x^2 + 16x\)
Section 4.3

Multiply: \(2x^2(3x^2 + 6x + 8)\)

\[
2x^2(3x^2 + 6x + 8) = \text{mono} \times \text{(any polynomial)}
\]

\[
(2x^2)(3x^2) + (2x^2)(-6x) + (2x^2)(8)
\]

\[
(2)(3) x^2 \cdot x^2 + (2)(-6) x^2 \cdot x + (2)(8) x^2
\]

\[
6x^4 - 12x^3 + 16x^2
\]

Answer: \(6x^4 - 12x^3 + 16x^2\)

C TYPE III: BINOMIALS × BINOMIALS

LAWS & PROCESSES

Multiply BINOMIAL × BINOMIAL

Everything in the 1\(^{\text{st}}\) parentheses goes to

Everywhere in the 2\(^{\text{nd}}\) parentheses

\[
(a + b)(c + d) =
\]

\[
a(c + d) + b(c + d) =
\]

\[
ac + ad + bc + bd
\]

1. Multiply each term in the 1\(^{\text{st}}\) set of parentheses by each term in the 2\(^{\text{nd}}\) set of parentheses.
2. Format it as in TYPE II: \((\text{mono})\times(\text{term}) + (\text{mono})\times(\text{term}) + \ldots\) keeping the sign of each term with it in the parentheses

EXAMPLES

Multiply: \((x + 3)(x - 7)\)

\[
(x + 3)(x - 7) = \text{binomial} \times \text{binomial}
\]

\[
x(x - 7) + 3(x - 7)
\]

\[
x^2 - 7x + 3x - 21
\]

\[
x^2 - 4x - 21
\]

Format as \((\text{mono})\times(\text{term}) + (\text{mono})\times(\text{term}) + \ldots\)

Multiply each term

Combine like terms to simplify

Answer: \(x^2 - 4x - 21\)
Multiply: \((4x - 2)(2x - 3)\)

\[
(4x - 2)(2x - 3) = 4x(2x - 3) - 2(2x - 3)
\]

\[
(4x)(2x) + (4x)(-3) + (-2)(2x) + (-2)(-3)
\]

\[
8x^2 - 12x - 4x + 6
\]

\[
8x^2 - 16x + 6
\]

Answer: \(8x^2 - 16x + 6\)

**ADDITIONAL INFO**

What is the F.O.I.L. Method?

The F.O.I.L. Method is a mnemonic device (tool for memorization). It is no different than the rule: **everything in the 1st parentheses goes to everything in the 2nd parentheses**

\[(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd\]

**F** = First, **O** = Outside, **I** = Inside, **L** = Last

In \((a + b)(c + d)\)...

First terms \(= a \cdot c\), Outside terms \(= a \cdot d\), Inside terms \(= b \cdot c\), and Last terms \(= b \cdot d\)

Hence, \(ac + ad + bc + bd\).

It is our recommendation that if you are familiar with the F.O.I.L. Method and comfortable with it, to use it. However, if you are new to F.O.I.L., the effort to memorize it may be more challenging than just applying the principle **Everything in the 1st parentheses goes to everything in the 2nd parentheses**.

At this point, it is good to remember that all multiplication of polynomials types come back to the simplicity of monomials \(\times\) monomials. A binomial \(\times\) binomial is just four monomial \(\times\) monomial problems in one. It is helpful to remember this in our final type. Remember also that the expression “any polynomial” refers to any polynomial with more terms than one.

**D** 

**TYPE IV: ANY POLYNOMIAL \(\times\) ANY POLYNOMIAL**

**LAWS & PROCESSES**

Multiply ANY POLYNOMIAL \(\times\) ANY POLYNOMIAL

Apply the same principle as a binomial \(\times\) binomial problem: **Everything in the 1st parentheses goes to everything in the second parenthesis.**

1. Multiply each term in the 1st set of parentheses by each term in the 2nd set of parentheses.
2. Format it as in TYPE II: \((\text{mono}) \times (\text{term}) + (\text{mono}) \times (\text{term}) + \ldots\)
   keeping the sign of each term with it in the parentheses.
10 Multiply: \((x - 2)(x^2 - 5x + 6)\)

\[
(x - 2)(x^2 - 5x + 6) = x(x^2 - 5x + 6) - 2(x^2 - 5x + 6)
\]

\[
(x)(x^2) + (x)(-5x) + (x)(6) + (-2)(x^2) + (-2)(-5x) + (-2)(6)
\]

\[
x^3 - 5x^2 + 6x - 2x^2 + 10x - 12
\]

\[
x^3 - 7x^2 + 16x - 12
\]

**Answer:** \(x^3 - 7x^2 + 16x - 12\)

11 Multiply: \((2x^2 - 5x + 1)(x^2 + 5x - 1)\) (trinomial \(\times\) trinomial)

\[
(2x^2 - 5x + 1)(x^2 + 5x - 1)
\]

\[
2x^2(x^2 + 5x - 1) - 5x(x^2 + 5x - 1) + 1(x^2 + 5x - 1)
\]

**first term** \((2x^2)(x^2) + (2x^2)(5x) + (2x^2)(-1)\)

**second term** \((-5x)(x^2) + (-5x)(5x) + (-5x)(-1)\)

**third term** \((1)(x^2) + (1)(5x) + (1)(-1)\)

\[
2x^4 + 10x^3 - 2x^2 - 5x^3 - 25x^2 + 5x + x^2 + 5x - 1
\]

\[
2x^4 + 10x^3 - 2x^2 - 5x^3 - 25x^2 + x^2 + 5x - 1
\]

\[
2x^4 + 5x^3 - 26x^2 + 10x - 1
\]

**Answer:** \(2x^4 + 5x^3 - 26x^2 + 10x - 1\)

The following section teaches **shortcuts**. It is critical to understand that none of these shortcuts can be applied to all **binomials \(\times\) binomials** problems – only the specific circumstances indicated. Everything learned in section 4.5 about how to multiply **binomials \(\times\) binomials** will yield correct answers to these problems without learning any of the shortcut methods. However, studying the shortcuts of these **binomial \(\times\) binomial** products can be helpful in understanding some of our work in our next chapter on factoring.
When multiplying polynomials, occasionally we’ll run into a \textbf{binomial \times binomial} that looks like these:

\[(x + 7)(x - 7) \quad (2x - 4)(2x + 4) \quad (x + y)(x - y)\]

Notice that each pair of \textbf{binomial \times binomial} has two things in common:

1. The two terms in the 1\textsuperscript{st} binomial match the two terms in the 2\textsuperscript{nd} binomial (except the sign)
2. One binomial is a sum (addition) and the other is a difference (subtraction).

Let’s multiply these three sets of binomials and look for a pattern (\textit{shortcut}) to the answers.

\[
\begin{align*}
12 \quad (x + 7)(x - 7) & \quad (2x - 4)(2x + 4) & \quad (x + y)(x - y) \\
(x)(x) + (x)(-7) + (7)(x) & \quad (2x)(2x) + (2x)(4) + (-4)(2x) & \quad (x)(x) + (x)(-y) + (y)(x) \\
+ (7)(-7) & \quad + (4)(4) & \quad + (y)(-y) \\
x^2 - 7x + 7x - 49 & \quad 4x^2 + 8x - 8x - 16 & \quad x^2 - xy + xy - y^2 \\
x^2 - 49 & \quad 4x^2 - 16 & \quad x^2 - y^2
\end{align*}
\]

We see a pattern:

1. The middle terms created by multiplying always cancel each other out.
2. The answer is always the \((1\textsuperscript{st} \text{term})^2 - (2\textsuperscript{nd} \text{term})^2\).

So here’s a summary of what you’ve found:

\begin{itemize}
  \item A Pattern for \textbf{BINOMIAL \times BINOMIAL in a SUM \times DIFFERENCE}
\end{itemize}

The product of the sum and difference of the same two terms will always follow the format:

\[ (A + B)(A - B) = A^2 - B^2 \]

where \(A\) is the 1\textsuperscript{st} term and \(B\) is the second term.

\begin{itemize}
  \item 1. \(A = 1\textsuperscript{st} \text{term. Square it.}\)
  \item 2. \(B = 2\textsuperscript{nd} \text{term. Square it.}\)
  \item 3. Set up answers as \(A^2 - B^2\)
\end{itemize}

Using the standard \textbf{binomial \times binomial} steps will yield the same answer as well.

Section 4.3
We see a pattern:

1. Each \((\text{binomial})^2 = \text{binomial} \times \text{binomial}\) ...see Common Mistakes Section below.
2. The answer is always a trinomial.
3. The trinomial answer is always the \((1^{st} \text{term})^2 \pm 2(1^{st} \text{term})(2^{nd} \text{term}) + (2^{nd} \text{term})^2\)
4. The \pm is determined by whether the binomial is a sum or a difference to begin with.
People often take \((x + 3)^2\) and incorrectly say the answer is \(x^2 + 9\)

1. They apply the Law of Exponent Power Rule from Section 4.1 which states that everything in the parentheses receives the power. This is only true for monomials, and we have a binomial.
2. When they do this they just square the \(x\) and square the 3 to get \(x^2 + 9\) —an incorrect answer
3. It is critical to remember that anything being squared is really that thing times itself, i.e. \((x + 3)^2 = (x + 3)(x + 3)\). Even the Power Rule is just a short cut around this fact, i.e. \((3x)^2 = (3x)(3x)\).

### A Pattern for BINOMIAL \(\times\) BINOMIAL in a \((BINOMIAL)^2\):

The product of a \((BINOMIAL)^2\) will always follow the format:

\[(A \pm B) = A^2 \pm 2AB + B^2\] where \(A\) is the 1\(^{\text{st}}\) term and \(B\) is the 2\(^{\text{nd}}\) term.

1. \(A = 1^{\text{st}}\) term. Square it.
2. \(B = 2^{\text{nd}}\) term. Square it.
3. Multiply \(2(A \cdot B)\).
4. Set up answer as \((A^2 \pm 2AB + B^2)\).
5. The \(\pm\) is determined by the sign of \(B\).

Using the standard \(\text{binomial} \times \text{binomial}\) steps will yield this same answer as well.

### EXAMPLES

<table>
<thead>
<tr>
<th>Example</th>
<th>Expression</th>
<th>Steps</th>
</tr>
</thead>
</table>
| 21      | \((x - 5)^2\) | \(A = x\); \(B = -5\)  
\[A^2 = x^2\]  
\[2(A \cdot B) = 2(-5x) = -10x\]  
\[A^2 + 2AB + B^2 = x^2 - 10x + 25\] |
| 22      | \((2x + y)^2\) | \(A = 2x\); \(B = y\)  
\[A^2 = 4x^2\]  
\[2(A \cdot B) = 2(2xy) = 4xy\]  
\[A^2 + 2AB + B^2 = 4x^2 + 4xy + y^2\] |

**Remember:** this only works for \((BINOMIAL)^2\). If you find memorizing this shortcut difficult you can always use your skills of multiplying a \(\text{binomial} \times \text{binomial}\) remembering how crucial it is to convert \((BINOMIAL)^2 = (BINOMIAL) \times (BINOMIAL)\)
Simplify.

1. \[\frac{kx^2g^{-1}}{x^{-3}f^2}\]
2. \[\left(\frac{1}{4a^2b^{-3}}\right)^{-1}\]

Perform the indicated operation. Write your answer in both scientific notation and decimal form.

3. \[1.23 \times 10^{-3} \cdot 4.36 \times 10^4\]
4. \[\frac{4.36 \times 10^3}{5.02 \times 10^{-2}}\]
5. \[\frac{6.02 \times 10^{23}}{4.8 \times 10^{25}}\]

Identify each term. Name the coefficient and degree of each term, as well as the degree of the polynomial.

6. \[4q^3 - 2q^2 + 3q - 2\]
7. \[3p^2 + 4p\]
8. \[3j^3 - 5\]

The area of the black circle is \[6y^2 - 2y + 3\]. The area of the trapezoid is \[-2y^2 + 4y + 11\]. Find the area of the white section between the black circle and the outer trapezoid.

9. 

Perform the indicated operations.

10. \[-2x + (3x - 4)\]
11. \[-(3x + 4) - (2x^2 + x)\]
12. \[3z^3 - (2z^2 + 7z^3) + \frac{1}{2}z^2\]
13. \[3m(k^2 - 2m^2 + 1)\]
14. \[(3s - 1)(s + 4)\]
15. \[(s^2 - 2)(s + 2)\]
16. \[(a + b)(2a^2 + a - 3)\]
17. \[(c^2 - 2c + 1)(2c^2 + c - 3)\]

Perform the indicated operations.

18. \[(x - 3)(x + 3)\]
19. \[(2x + 1)(2x - 1)\]
20. \[(3mn - 1)(3mn + 1)\]
21. \[(3a + 4b)(3a - 4b)\]
22. \[(k^3 - 3)(k^3 + 3)\]
23. \[(3x + 1)^2\]
24. \[(2x - 1)^2\]
25. \[(k + 2)^2\]
26. \[(z^2 - 1)^2\]
27. \[(k^3 + 2m)^2\]

Preparation:

28. \[\frac{(8y^2 + 4)}{2}\]
29. \[(6x^3 - 2x^2 + x) \div x\]
Answers:

1. \( \frac{kx^5 g}{f^3} \)
2. \( \frac{4a^2}{b^3} \)
3. \( 5.3628 \times 10^1, \ 53.628 \)
4. \( 8.685 \times 10^4, \ 86852.59 \)
5. \( 1.254 \times 10^{-2}, \ .01254 \)
6. \( 4q^3: \text{coefficient} = 4, \text{degree} = 3; \)
   \( -2q^2: \text{coefficient} = -2, \text{degree} = 2 \)
   \( 3q: \text{coefficient} = 3, \text{degree} = 1 \)
   \( -2: \text{coefficient} = -2, \text{degree} = 0 \)
   degree of polynomial = 3
7. \( 3p^2: \text{coefficient} = 3, \text{power} = 2 \)
   \( 4p: \text{coefficient} = 4, \text{power} = 1 \)
8. \( 3j^3: \text{coefficient} = 3, \text{power} = 3 \)
   \( -5: \text{coefficient} = -5, \text{power} = 0 \)
9. \( -8y^2 + 6y + 8 \)
10. \( x - 4 \)
11. \( -2x^2 - 4x - 4 \)
12. \( -4z^3 - \frac{3}{2}z^2 \)
13. \( -6m^3 + 3mk^2 + 3m \)
14. \( 3s^2 + 11s - 4 \)
15. \( s^3 + 2s^2 - 2s - 4 \)
16. \( 2a^3 + a^2 - 3a + 2a^2b + ab - 3b \)
17. \( 2c^4 - 3c^3 - 3c^2 + 7c - 3 \)
18. \( x^2 - 9 \)
19. \( 4x^2 - 1 \)
20. \( 9m^2n^2 - 1 \)
21. \( 9a^2 - 16b^2 \)
22. \( k^6 - 9 \)
23. \( 9x^2 + 6x + 1 \)
24. \( 4x^2 - 4x + 1 \)
25. \( k^2 + 4k + 4 \)
26. \( z^4 - 2z^2 + 1 \)
27. \( k^6 + 4k^3 m + 4m^2 \)
28. In class.
29. In class.
In order to accept Step 1 as a logical manipulation of numbers to look different, yet remain the same, we must accept the fact that \( \frac{A+B}{C} = \frac{A}{C} + \frac{B}{C} \).

For example, \( \frac{8+2}{2} = \frac{10}{2} = 5 \) while \( \frac{8}{2} + \frac{2}{2} = 4 + 1 = 5 \). Hence, \( \frac{8+2}{2} = \frac{8}{2} + \frac{2}{2} = 5 \).

**EXAMPLES**

1. **Divide:** \( \frac{35x^5 - 20x^3}{5x^3} \)

   Step 1: Place the monomial under each term in the polynomial
   
   \[
   \frac{35x^5}{5x^3} - \frac{20x^3}{5x^3}
   \]

   Step 2: Simplify each new term. Divide numbers with numbers and like variables with like variables.

   \[
   \frac{35}{5}x^{5-3} - \frac{20}{5}x^{3-3}
   \]

   Using the quotient rule (4.1) we subtract top-bottom exponents

   \[7x^2 - 4x^0\]

   The zero rule says \( x^0 = 1 \) and we know anything times 1 = ‘s itself

   **Answer:** \( 7x^2 - 4 \)
Divide: \((24a^6 - 48a^5 + 10a^4) \div (4a^4)\)

\[
\frac{24a^6}{4a^4} - \frac{48a^5}{4a^4} + \frac{10a^4}{4a^4}
\]

Step 1: Place the monomial under each term in the polynomial.

\[
\frac{24}{4}a^{6-4} - \frac{48}{4}a^{5-4} + \frac{10}{4}a^{4-4}
\]

Step 2: Simplify each new term. Divide numbers with numbers and like variables with like variables.

\[
6a^2 - 12a + \frac{5}{2}a^0
\]

Not all fractions cancel out – reduce them as much as possible.

\[
6a^2 - 12a + \frac{5}{2}
\]

In simplifying, use the zero and one rule for exponents if needed.

Answer: \(6a^2 - 12a + \frac{5}{2}\)

Divide: \(\frac{15x^{10}y^7 - 8x^6y^3 + 18x^4y - 3x^2y}{3x^2y}\)

\[
\frac{15x^{10}y^7}{3x^2y} - \frac{8x^6y^3}{3x^2y} + \frac{18x^4y}{3x^2y} - \frac{3x^2y}{3x^2y}
\]

Step 1: Place the monomial under each term in the polynomial.

\[
\frac{15}{3}x^{10-2}y^{7-1} - \frac{8}{3}x^{6-2}y^{3-1} + \frac{18}{3}x^{4-2}y^{1-1} - \frac{3}{3}x^{2-2}y^{1-1}
\]

Step 2: Simplify each new term. Divide numbers with numbers and like variables with like variables.

\[
5x^8y^6 - \frac{8}{3}x^4y^2 + 6x^2y^1 - 1
\]

Not all fractions reduce completely

Use the Zero and One Rule to finish up

Answer: \(5x^8y^6 - \frac{8}{3}x^4y^2 + 6x^2 - 1\)
4.4 EXERCISE SET

Simplify.

4.1  
1. \(a^2 e^3 i^{-2} o^0 u^{-2} a^3 e^{-4} i^{-2} o^{-2} u^4\)
2. \(e^2 i^{-3} e^{-4} i^{-2} o^3\)
3. \(\frac{r^2 a^{-3} e^2}{c^4 a r^{-1}}\)

Simplify. Write your answer in descending order.

4. \(\frac{3}{(s+1)^{-2}}\)
5. \(-j + 4j^2 - 3j - 1 + \frac{1}{j^{-1}}\)

Find a polynomial that describes the perimeter of the shape.

4.2  
6.

Find a polynomial that describes the area or volume of these shapes.

4.3  
7. 

8. 

9. 

10. 

Perform the indicated operations

11. \(4x + 2x^2 + x - 2x^2 + 5x - 3x^2\)
12. \(z^3 + 4z^2 - 12z - (-z^3 + z - 3z^2)\)
13. \(4y + 3y^2 x + y(4yx + 5)\)
14. \(-12m(6m^3 - 8m^2 + m - 12)\)
15. \(8g(3h^3 + 9h - 12)\)
16. \((3p + 4)(p - 4)\)
17. \((2l + 4)(l + j)\)
18. \((4k - 1)(4k^2 - k + 6)\)
19. \((7a^2 + 3)(2a^2 + 3a + 6)\)
20. \(\left(\frac{1}{2} b + 1\right)\left(\frac{1}{2} b - 1\right)\)
21. \((8f^2 + 6)(8f^2 - 6)\)
22. \((m^2 n^2 + 1)(m^2 n^2 - 1)\)
23. \((7x + 1)^2\)
24. \((w^2 x^2 - y^2 z^2)^2\)
Perform the indicated operations.

25. \(9m^4 \div 3m\)

26. \((6x^2 + 12x) \div 3x\)

27. \((2x^2y + 3xy + y^2 - 2) \div 2x\)

28. \((-56y^4 + 44y^3 + 64y^2 - 16y) \div 8y\)

29. \((4b^{13} - 9b^8 - 3b^5) \div 3b^3\)

30. \((-6y^5 - 3y^3 + y) \div 2y\)

Preparation:
Is there anything you can factor out of the numerator to make these problems easier?

31. \(\frac{18x^2 + 24x - 9}{3}\)

32. \(\frac{7x^2y^3 + 3x^2y^2z - 2x^2yz^2 + 5x^2z^3}{x^2}\)
Answers:

1. \( \frac{a^2u^2}{e^4a^2} \)
2. \( \frac{e^6o^3}{l^5} \)
3. \( \frac{r^4e^2}{a^4c^2} \)
4. \( 3s^2 + 6s + 3 \)
5. \( 4j^2 - 3j - 1 \)
6. \( s^3 - \frac{1}{2}s^2 + s + \frac{9}{2} \)
7. \( 12x^2 - 10x - 8 \)
8. \( x^4 + 4x^3 + 3x^2 - 4x - 4 \)
9. \( 4z^4 - 16z^2 \)
10. \( \pi(x^3 - 6x^2 + 9x) \) or \( \pi x^3 - 6\pi x^2 + 9\pi x \)
11. \( -3x^2 + 10x \)
12. \( 2z^3 + 7z^2 - 13z \)
13. \( 7y^2x + 9y \)
14. \( -72m^4 + 96m^3 - 12m^2 + 144m \)
15. \( 24gh^3 + 72gh - 96g \)
16. \( 3p^2 - 8p - 16 \)
17. \( 2l^2 + 2lj + 4l + 4j \)
18. \( 16k^3 - 8k^2 + 25k - 6 \)
19. \( 14a^4 + 21a^3 + 48a^2 + 9a + 18 \)
20. \( \frac{1}{4}b^2 - 1 \)
21. \( 64f^4 - 36 \)
22. \( m^4n^4 - 1 \)
23. \( 49x^2 + 14x + 1 \)
24. \( w^4x^4 - 2w^2x^2y^2z^2 + y^4z^4 \)
25. \( 3m^3 \)
26. \( 2x + 4 \)
27. \( xy + \frac{3}{2}y + \frac{y^2}{2x} - \frac{1}{x} \)
28. \( -7y^3 + \frac{11}{2}y^2 + 8y - 2 \)
29. \( \frac{4}{3}b^{10} - 3b^5 - b^2 \)
30. \( -3y^4 - \frac{3}{2}y^2 + \frac{1}{2} \)
31. In class.
32. In class.
## 4.1 Exponents and Their Laws

### ZERO AND ONE RULE
- **Anything to the power of zero** = 1 (if \( a \neq 0 \))
- **Anything to the power of one** = itself

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^0 = 1 )</td>
<td>( 4^0 = 1 )</td>
<td></td>
</tr>
<tr>
<td>( a^1 = a )</td>
<td>( 4^1 = 4 )</td>
<td></td>
</tr>
</tbody>
</table>

### PRODUCT RULE
- Exponents being multiplied with the same base are added.

\[
a^m \cdot a^n = a^{m+n}
\]

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^3 x^2 = x^5 )</td>
<td>( x^4 x^{-2} = x^2 )</td>
<td></td>
</tr>
<tr>
<td>( 2^3 \cdot 2^4 = 2^7 )</td>
<td>( 2^{-3} \cdot 2^5 = 2^2 )</td>
<td></td>
</tr>
</tbody>
</table>

### QUOTIENT RULE
- Exponents being divided with the same base are subtracted. (if \( a \neq 0 \))

\[
\frac{a^m}{a^n} = a^{m-n}
\]

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x^4}{x^1} = x^3 )</td>
<td>( \frac{x^3}{x^8} = x^{-5} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3^6}{3^2} = 3^4 )</td>
<td>( \frac{6^5}{6^{12}} = 6^{-7} )</td>
<td></td>
</tr>
</tbody>
</table>

### NEGATIVE EXPONENT RULE
- All negative exponents can be converted to a positive exponent by simply taking the reciprocal.

\[
a^{-m} = \frac{1}{a^m}
\]

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^{-2} = \frac{1}{x^2} )</td>
<td>( \frac{1}{x^{-3}} = x^3 )</td>
<td></td>
</tr>
<tr>
<td>( 4^{-3} = \frac{1}{4^3} )</td>
<td>( \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 )</td>
<td></td>
</tr>
</tbody>
</table>

### POWER RULE
- To raise a power to a power, we multiply the exponents. Every base in the parentheses ( ) receives the power.

\[(a^m)^n = a^{mn}\]

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x^{-2})^{-1} = x^2 )</td>
<td>( (x^2)^3 = x^6 )</td>
<td></td>
</tr>
<tr>
<td>( (3^{-4})3^3 = 3^{-12} )</td>
<td>( (3^4)^2 = 3^8 )</td>
<td></td>
</tr>
</tbody>
</table>

### Simplifying Questions
1. **Do I have any exponents of zero and one?**
2. **Do I have any products and quotients with the same base?**
3. **Are there any negative exponents?**
4. **Are there any powers being raised to powers?**

These same rules apply when you are multiplying or dividing with scientific notation:

### Steps of Multiplying and Dividing Sci. Notation
1. Combine number with numbers (\( \times \) or \( \div \))
2. Combine base 10 with base 10 (\( \times \) or \( \div \) using laws of exponents)
3. Write answer in scientific notation if it’s not already
Principles About Terms

1. Terms are separated by plus (+) and minus (-) signs.
   a. Hence $7x - 5y$ has two terms, and $x^2 + 5x - 6$ has three terms.

2. The sign before the term always goes with it.
   a. Hence the two terms in $7x - 5y$ are $7x$ and $-5$, and the three terms in $x^2 + 5x - 6$ are $x^2$, $5x$, and $-6$.

A. Terms are broken into 3 parts; the coefficient, the variable(s), and the exponent(s).

   \[ 5x^3 - 2x^2 + 1x^1 \]

B. Like terms are defined as having the same number of variables and are added or subtracted using coefficients.

   \[ 3x^2 + 2x^2 = 5x^2 \quad 4x^2y - 2xy^2 + x^2y = 5x^2y - 2xy^2 \]

Adding and Subtracting Polynomials

1. Distribute parentheses. In addition, parentheses can just be dropped. In subtraction, the minus sign becomes a $(-1)$ distributor.
2. Combine like terms.

When adding or subtracting polynomials be sure the sign is properly distributed before combining like terms.

\[
(3x^5 - 2x^3 + 1) + (2x^5 - 3x^3 - x) = 3x^5 - 2x^3 + 1 + 2x^5 - 3x^3 - x
\]

\[
(4y^3 + 2y^2 - 1) - (4y^3 + y^2 - 3) = 4y^3 + 2y^2 - 1 - 4y^3 - y^2 + 3
\]

4.3 Multiplying Polynomials

A. Monomials $\times$ Monomials: Multiply numbers with numbers and like variables with like variables.

   \[ 3x^2y \cdot 3xz^2 = 9x^3yz^2 \]

B. Monomials $\times$ Polynomials: Distribute the monomial in by multiplying it to each monomial inside of the polynomial.

\[ 3x(x^3 + 2x^2 - 4x + 1) = 3x^4 + 6x^3 - 12x^2 + 3x \]
C. Binomials × Binomials: Multiply each term in the 1st ( ) by each term in the 2nd ( ).

\[(2x + 3)(3x + 5)\]

\[6x^2 + 10x + 9x + 15\]

D. Binomials × Polynomials: Multiply each term in the 1st ( ) by each term in the 2nd ( ).

\[(-x + 2)(3x^2 + 2x - 4)\]

\[-3x^2 - 2x^2 + 4x + 6x^2 + 4x - 8\]

**Special Products**

\[
\begin{array}{|c|c|}
\hline
\text{(sum) × (difference)} & \text{(binomial)²} \\
\hline
(A + B)(A - B) = A^2 - B^2 & (A + B)² = A² + 2AB + B² \\
(3x - y)(3x + y) & (7x² - 2y)² \\
9x² - y² & 49x⁴ - 28x²y + 4y² \\
\hline
\end{array}
\]

Note: The answer will always be in the form \(A^2 - B^2\).

Note. The answer will always be in the form \(A^2 + 2AB + B²\).

**4.4 Polynomials: Division**

A. Polynomials ÷ Monomials: Divide each monomial within the polynomial by the monomial.

\[
(3x^2 - 6x + 12) ÷ 3x
\]

\[
\frac{3x^2}{3x} - \frac{6x}{3x} + \frac{12}{3x}
\]

\[x - 2 + \frac{4}{x}\]
Chapter 4 Review 1

Simplify or evaluate.

1. \(3^2 \cdot 3^{-4}\)
2. \((\frac{2}{3})^2\)
3. \(\frac{m^{-2}a^2t^{-3}h^{-3}}{h^2a^{-7}p^2p^{-3}y^2}\)
4. \(\frac{g^4m^3m^{-2}n^5}{g^2(agn^2)^{-1}g^3n(n^2)^3ma}\)
5. \(\left(\frac{4xy^3}{3x^2y^{-5}z^3}\right)^{-2}\)
6. \(\left(\frac{2a^{-5}b^4c^{-1}}{a^{-2}b^9}\right)^4\)

Perform the operation and write your answer in scientific form. Round to three decimal places.

7. \(\frac{6.3781 \times 10^3}{1.989 \times 10^{30}}\)
8. \(\frac{1.51 \times 10^{-7}}{5.002 \times 10^{-5}}\)

Evaluate the polynomials at the given values.

9. \(a^2 - 3a + 2\), for \(a = 5\)
10. \(x^3 + 2x^2 - 4x\), for \(x = -3\)

Add or subtract the polynomials. Write answer in descending order.

11. \((-3y^2 + 7) + (y^2 + 2y - 6)\)
12. \((4x^3 - 1 + x + 3x^2) + (x^2 - x + 5)\)
13. \((3j^4 - j + 2j^2) - (-j^2 + 4j + j^4)\)
14. \((-3p - 9p^2) - (-12p^2 - 5p + 4)\)

Perform the indicated operations.

15. \((3x)(x - 5)\)
16. \(\left(-\frac{1}{2}a\right)(4a^2 + 6a - 2)\)
17. \((5d^2 - 1)(3d + 1)\)
18. \((6e + 4)(-5e + 3)\)
19. \((g - 7)(g + 6)\)
20. \((hi + 2)(i - 3h)\)
21. \((j + 2k)(k^2 - 2j)\)
22. \((2l - 3)(2l + 3)\)
23. \((4m^3 - 2)(4m^3 + 2)\)
24. \((n^6 + 3o^3)(n^6 - 3o^3)\)
25. \((5p^2 + 1)^2\)
26. \((-q + 2)^2\)
27. \((2r + 2s)^2\)
28. \((t + 3)(t^2 - 3t - 4)\)
29. \((u^2 + u^3)(u^3 - u^2 + u - 1)\)
30. \((2x^4 + 7x^3 - x)(x^2 + 3x + 2)\)

Divide the polynomials.

31. \((-80w^6 + 35w^5 - 50w^4) \div 10w^4\)
32. \((33x^3 - 18x^2 + 3x) \div 3x\)
33. \((24y^3 - 2y^2) \div 2y\)
34. \((8abz^3 - 2jz^4 + z^5) \div z^3\)
Answers:

1. \(\frac{1}{9}\)
2. \(\frac{4}{9}\)
3. \(\frac{a^9 p}{m^2 t^3 h^5 y^2}\)
4. 1
5. \(\frac{9x^2 z^6}{16y^{16}}\)
6. \(\frac{16}{a^{12} b^{20} c^4}\)
7. \(3.207 \times 10^{-27}\)
8. \(3.019 \times 10^{-3}\)
9. 12
10. 3
11. \(-2y^2 + 2y + 1\)
12. \(4x^3 + 4x^2 + 4\)
13. \(2j^4 + 3j^2 - 5j\)
14. \(3p^2 + 2p - 4\)
15. \(3x^2 - 15x\)
16. \(-2a^3 - 3a^2 + a\)
17. \(15d^3 + 5d^2 - 3d - 1\)
18. \(-30e^2 - 2e + 12\)
19. \(g^2 - g - 42\)
20. \(hi^2 - 3h^2 i + 2i - 6h\)
21. \(2k^3 + jk^2 - 4jk - 2j^2\)
22. \(4l^2 - 9\)
23. \(16m^6 - 4\)
24. \(n^{12} - 9o^6\)
25. \(25p^4 + 10p^2 + 1\)
26. \(q^2 - 4q + 4\)
27. \(4r^2 + 8rs + 4s^2\)
28. \(t^3 - 13t - 12\)
29. \(u^6 - u^2\)
30. \(2x^6 + 13x^5 + 25x^4 + 13x^3 - 3x^2 - 2x\)
31. \(-8w^2 + \frac{7}{2}w - 5\)
32. \(11x^2 - 6x + 1\)
33. \(12y^2 - y\)
34. \(8ab - 2jz + z^2\)
Chapter 4 Review 2

Simplify or evaluate.

1. \(5^{-5} \cdot 5^2\)
2. \(\left(\frac{5}{4}\right)^{-2}\)
3. \(\frac{h^{-3}o^2 p^4 e^{-2}}{o^2 b^{-2} e^2 d^3 i^{-1} e^{6} n^2 c^{-3} e^{-4}}\)
4. \(\frac{n^3 a^{-3} p^2 s^6}{a^{-3} s^6 p^2 o^{-3} o^2 n^{-2}}\)
5. \(\left(\frac{x^{-3} y^7 z^{-4}}{5x^2 y^{8} z^3}\right)^{-3}\)
6. \(\left(\frac{3l^2 m^{-3} n^{-1}}{m^4 n^{-2}}\right)^5\)

Perform the operation and write your answer in scientific form. Round to three decimal places.

7. \(\frac{3.99 \times 10^6}{5.22 \times 10^{-4}}\)
8. \(\frac{7.031 \times 10^{-7}}{9.201 \times 10^{-5}}\)

Evaluate the polynomials at the given values.

9. \(a^2 - 3a + 2\), for \(a = 2\)
10. \(-x^3 + 4x^2 - x\), for \(x = -2\)

Add or subtract the polynomials. Write answer in descending order.

11. \((-y^2 + 5) + (3y^2 - 2y - 6)\)
12. \((3m^2 + 7m^3 - m) + (8m - 6m^3)\)
13. \((3z^4 - z + 2z) - (-3z^2 + z + 2z^3)\)
14. \((-x - 9x^2) - (-2x^2 - 5x + 3)\)

Perform the indicated operations.

15. \((f + 1)(f - 3)\)
16. \((g - 2)(g + 6)\)
17. \((-h^2 + 4h)(-2h^2 - 3h)\)
18. \((3l^3 - 2i)(4i^2 + 1)\)
19. \((j + 8)(j - 8)\)
20. \((2k + 4)(2k - 4)\)
21. \((7l^{10} - 5m^{12})(7l^{10} + 5m^{12})\)
22. \((6n^2 - 1)^2\)
23. \((11o + 3)^2\)
24. \(4p(p^2 - 1)(p^2 + 1)\)
25. \(2q(q + 6)^2\)
26. \((r + 3)(r^2 - 2r + 3)\)
27. \((s^2 - 1)(s^4 - 2s^2 + 5)\)
28. \((u^2 + 1)(v^2 - 1)\)
29. \((7x - 4)^2\)
30. \((w^2 + x^2)(y + z^2)\)

Divide the polynomials.

31. \((5k^2 - 35k) ÷ 5k\)
32. \((e^4 - e^3 + 3e^2 - 2e) ÷ 4e\)
33. \((42l^2 + 21l - 72) ÷ 6\)
34. \((39l^9 - 65l^8 + 52l^7 + 26l^4) ÷ 13l^3\)
Answers:

1. $\frac{1}{125}$
2. $\frac{16}{25}$
3. $\frac{b^2ic^3p^4}{h^3od^3n^2}$
4. $n^5o$
5. $\frac{125x^{15}z^{21}}{y^{45}}$
6. $\frac{243l^{15}n^5}{m^{35}}$
7. $7.644 \times 10^8$
8. $7.642 \times 10^{-3}$
9. 0
10. 26
11. $2y^2 - 2y - 1$
12. $m^3 + 3m^2 + 7m$
13. $3z^4 - 2z^3 + 3z^2$
14. $-7x^2 + 4x - 3$
15. $f^2 - 2f - 3$
16. $g^2 + 4g - 12$
17. $2h^4 - 5h^3 - 12h^2$
18. $12l^5 - 5l^3 - 2l$
19. $j^2 - 64$
20. $4k^2 - 16$
21. $49l^{20} - 25m^{24}$
22. $36n^4 - 12n^2 + 1$
23. $121o^2 + 66o + 9$
24. $4p^5 - 4p$
25. $2q^3 + 24q^2 + 72q$
26. $r^3 + r^2 - 3r + 9$
27. $s^6 - 3s^4 + 7s^2 - 5$
28. $u^2v^2 - u^2 + v^2 - 1$
29. $49x^2 - 56x + 16$
30. $w^2y + w^2z^2 + x^2y + x^2z^2$
31. $k - 7$
32. $\frac{1}{4}e^3 - \frac{1}{4}e^2 + \frac{3}{4}e - \frac{1}{2}$
33. $7l^2 + \frac{7}{2}l - 12$
34. $3l^6 - 5l^5 + 4l^4 + 2l$
Chapter 5:
FACTORIZING: THE FIVE METHODS

Overview

5.1 Intro to Factoring (Methods 1 & 2)
5.2 More Factoring (Method 3)
5.3 Factoring (Method 4)
5.4 Factoring (Special Cases)
5.5 Factoring: A holistic approach
5.6 Solving Equations by Factoring; Word Problems
FACTOR: Factors are numbers that can multiply together to get another number.

Example: 2, 3, and 5 are all factors of 30 because $2 \times 3 \times 5 = 30$. At the same time, we could say that 10 is a factor of 30 because $10 \times 3 = 30$. So if you want to know the factors of 30, the answer is any whole number that will multiply by another number to get 30. They are: 1, 2, 3, 5, 6, 10, 15, &/u000a30, because each of these factors can get to 30 by multiplying by another factor.

TO FACTOR A POLYNOMIAL: Factors do not have to be monomials. In Chapter 4 we learned about multiplying polynomials – multiplying two (or more) polynomials to get another polynomial. The polynomials you multiplied together are factors of the final polynomial. To factor a polynomial means to break it up into the pieces that can be multiplied together to be that polynomial.

Example: The polynomial $(6x^2 - 9x)$ can be factored into two smaller pieces: $(3x)(2x - 3)$. So $(3x)$ and $(2x - 3)$ are both factors of $(6x^2 - 9x)$. If you multiply the two factors, they will equal the original polynomial.

### THE FIVE METHODS OF FACTORING:

**Method** | **Type of Polynomial used on:**
--- | ---
1: Pull Out the Common Factor (C.F.) | All
2: Grouping | 4 Terms
3: $ax^2 + bx + c$, where $a = 1$ | Trinomials, where $a = 1$
4: $ax^2 + bx + c$, where $a \neq 1$ | Trinomials, where $a \neq 1$
5: Special Cases | Binomials & Trinomials
Before we learn the methods of factoring let’s practice identifying what methods are possibilities for the following polynomials by using the chart above. Just look at the type of polynomial as see which method we should try using to factor it.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Factoring Methods Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y^3 + 4y^2 + 2y + 8)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 2: Grouping</td>
</tr>
<tr>
<td>(x^2 + 6x + 9)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 3: (ax^2 + bx + c), where (a=1)</td>
</tr>
<tr>
<td>(16x^2 - 16x + 4)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 4: (ax^2 + bx + c), where (a\neq 1)</td>
</tr>
<tr>
<td>(54x^3 - 6x)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 5: Special Cases (because it’s a binomial)</td>
</tr>
<tr>
<td>(x^2 - 5x + 6)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 3: (ax^2 + bx + c), where (a=1)</td>
</tr>
<tr>
<td>(2c^3 + 8c^2 - 6c - 12)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 2: Grouping</td>
</tr>
<tr>
<td>(36a^2 - 25)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 5: Special Cases (because it’s a binomial)</td>
</tr>
<tr>
<td>(35x^3 + 42x^2 - 14x - 77xy - 14y + 7)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>(only method because it has 6 terms)</td>
</tr>
<tr>
<td>(6y^2 + 25y + 25)</td>
<td>Method 1: Pull out common factor</td>
</tr>
<tr>
<td></td>
<td>Method 4: (ax^2 + bx + c), where (a\neq 1)</td>
</tr>
</tbody>
</table>

This does not mean each of these methods will work, but by doing this we see what methods we need to explore for each polynomial, and which ones we don’t. Note how we applied method 1 to everything because it can be tried on all types of polynomials.

**DEFINITIONS & BASICS**

**CHECKING ANSWERS:** In one sense, factoring is the opposite of multiplication, and so when we achieve an answer in factoring we can multiply it out to check its validity.

**Example:** If asked to factor \(x^2 + 6x + 9\) and we get the answer \((x + 3)(x + 3)\), we can then multiply the answer \((x + 3)(x + 3)\) to check if it equals the original polynomial \(x^2 + 6x + 9\).

\[
(x + 3)(x + 3) = (x)(x) + (x)(3) + (3)(x) + (3)(3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9
\]

If the answer doesn’t match the original problem, then either your factoring or your check is incorrect.

**PRIME:** Not every polynomial is factorable. When a polynomial cannot be factored using any of the five methods, we say it is prime or unfactorable.
The first method you should try on EVERY factoring problem is method 1, pulling out the common factor. This will simplify the polynomial before you try any other methods.

**COMMON FACTOR:** A factor that two or more terms share in common.

**Example:** Two terms, 24 and 36, have the following factors:
- 24: 1, 2, 3, 4, 6, 8, 12, 24
- 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Common Factors: 1, 2, 3, 4, 6, & 12

**Example:** Two terms, $5x^2$ and $15x$, have the following factors:
- $15x^2$: 1, 5, $x$, $x^2$
- $15x$: 1, 3, 5, 15, $x$

Common factors: 1, 5, & $x$

**GREATEST COMMON FACTOR:** The biggest factor that ALL terms share in common. From here on we will refer to this as the “C.F.” It is the greatest numerical value and greatest variable combination they share in common. When they share both variables and numbers in common, the C.F. is expressed as a combination of both.

**Example:** From the above example, the C.F. (greatest common factor) of 24 and 36 is 12.

**Example:** From the above example, the C.F. of $5x^2$ and $15x$ is $5x$.

**Factoring Method 1:**
**Pull Out the Common Factor**

1. Identify the greatest common factor (C.F.) that ALL of the terms in the polynomial share in common.
2. Place the C.F. under each term in the polynomial.
3. Simplify each term that is now in the form: \( \frac{\text{MONOMIAL}}{\text{C.F.}} \)
4. Write answer in this format: C.F. (Simplified Polynomial)

**REMEMBER:** On which types of polynomial do we try method 1? Every polynomial. This important method needs to be considered in every factoring problem.
**Section 5.1**

**Step 1:** Identify the C.F. that **ALL** terms share

\[
\frac{3a}{3} - \frac{6b}{3}
\]

\[a - 2b\]

\[3(a - 2b)\]

**Step 2:** Place the C.F. under each term in the polynomial

\[
\frac{4x}{x} - \frac{5xy}{x}
\]

\[4 - 5y\]

**Step 3:** Simplify each new term

\[4 - 5y\]

**Step 4:** Answer format

\[x(4 - 5y)\]

---

**What if I don’t get the C.F. correct?** It’s not a big deal. It just means you will have to keep pulling out factors until you get everything out. It’s a little slower than getting the Greatest Common Factor out first, but it works.

**What if two out of three terms has a common factor?** It’s not a common factor unless it is common to **ALL** terms. You can only factor out something if every term shares it.

**What happens when terms cancel each other out?** Sometimes we think that if something cancels out, it equals zero, but check out this example:

**Example:** In example 4, the C.F. = 7xy. When we divided each term by the C.F. we saw this:

\[
\frac{21x^2y}{7xy} + \frac{35xy^2}{7xy} - \frac{7xy}{7xy}
\]

The last term, \(\frac{7xy}{7xy}\), does not disappear, it equals 1.
**Factoring Method 2: Grouping**

1. Check for **method 1** first (Pull out C.F.)
2. Break the 4 terms into a (binomial) + (binomial)
3. Pull out the C.F. from each binomial to get C.F. (binomial) ± C.F. (binomial)
4. If the two (binomial)’s are identical, then they are now a common factor, so you can pull that out to get (binomial)(C.F. ± C.F.).

**REMINDER:** On which types of polynomial do we use method 2? Only polynomials with **four terms**. However, remember to do method 1 (pull out C.F.) before you try method 2.

### EXAMPLES

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>STEPS</th>
<th>EXTRA HELP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^3 + 3x^2 + 4x + 12)</td>
<td><strong>Step 1:</strong> Check for C.F. (none in this case) <strong>Step 2:</strong> Break the 4 terms into a (binomial) + (binomial) <strong>Step 3:</strong> Pull out the C.F. from each binomial to get C.F. (binomial) ± C.F. (binomial) <strong>Step 4:</strong> The two binomials are now identical, so they are a C.F. of the whole polynomial. You can factor the binomial out with the left over C.F.’s forming another binomial. <strong>Answer format:</strong> ((\text{Binomial})x(\text{Binomial}) = (\text{C.F. Binomial})(\text{left over C.F.’s}))</td>
<td><strong>Most students are fine up through the point of pulling out the C.F. from each binomial where we get:</strong> (x^2(x + 3) + 4(x + 3)) At this point, it can help to let ((x + 3) = A), so we have: (x^2A + 4A) When we pull out a Common Factor of (A) we get: (\frac{x^2A + 4A}{A} = \frac{x^2A}{A} + \frac{4A}{A} = \frac{x^2A}{A} + 4) Now if we remember that (A = (x + 3)) we can say... (A(x^2 + 4) = (x + 3)(x^2 + 4))</td>
</tr>
<tr>
<td>((x^3 + 3x^2) + (4x + 12))</td>
<td>C.F. = (x^2) C.F. = (4)</td>
<td></td>
</tr>
<tr>
<td>(\left(\frac{x^3 + 3x^2}{x^2}\right) + \left(\frac{4x + 12}{4}\right))</td>
<td>(x^2(x + 3) + 4(x + 3))</td>
<td></td>
</tr>
<tr>
<td>(x^2(x + 3) + 4(x + 3))</td>
<td>(C.F.) (left over) (x + 3)(x^2 + 4))</td>
<td></td>
</tr>
</tbody>
</table>
### PROBLEM 6

**Problem:**

\[7xy + 28x + y + 4\]

<table>
<thead>
<tr>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Check for C.F. (none in this case)</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Break the 4 terms into a (binomial) + (binomial)</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Pull out the C.F. from each binomial to get C.F. (binomial) ± C.F. (binomial)</td>
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<tr>
<td><strong>Step 4:</strong> The two binomials are now identical, so they are a C.F. of the whole polynomial. You can factor the binomial out with the left over C.F.’s forming another binomial.</td>
</tr>
</tbody>
</table>

**Answer format:**

\[(\text{Binomial}) \times (\text{Binomial}) = (\text{C.F. Binomial})(\text{left over C.F.’s})\]

You can check by multiplying.

**Extra Help**

In Step 3: when \((y + 4)\) cancel each other out, it doesn’t just cease to exist — **it equals 1**

Hence, what’s left over in step 3 after pulling out the common parentheses of \((y + 4)\)

\[
\frac{7x(y + 4)}{(y + 4)} + \frac{(y + 4)}{(y + 4)}
\]

is 7x and 1, which now drop into the 2nd ( ) in the Step 4 answer format:

\[(y + 4)(7x + 1)\]

### PROBLEM 7

**Problem:**

\[2x^3 + 10x^2 - 3x - 15\]

<table>
<thead>
<tr>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Check for C.F. (none in this case)</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Break the 4 terms into a (binomial) + (binomial)</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Pull out the C.F. from each binomial to get C.F. (binomial) ± C.F. (binomial)</td>
</tr>
<tr>
<td><strong>Step 4:</strong> The two binomials are now identical, so they are a C.F. of the whole polynomial. You can factor the binomial out with the left over C.F.’s forming another binomial.</td>
</tr>
</tbody>
</table>

**Answer format:**

\[(\text{Binomial}) \times (\text{Binomial}) = (\text{C.F. Binomial})(\text{left over C.F.’s})\]

You can check by multiplying.

**Extra Help**

**Note:** When we broke the 4 terms into a (Bi) + (Bi) we took the signs of each term with them and left a + between the two parentheses:

\[(2x^3 + 10x^2) + (-3x - 15)\]

This helps us keep track of our negative #’s.

**Note:** In order for factoring by grouping to work, after we pull out the C.F. in step 2, each parentheses must be **EXACTLY** the same. This happened because we pulled out a C.F. of -3 rather than just 3. Had we not done this, the two parentheses would be different:

\[(x + 5)\text{ and }(-x - 5)\]

Sometimes it is necessary to pull out a negative sign with the C.F to make the parentheses the same.
<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>STEPS</th>
<th>EXTRA HELP</th>
</tr>
</thead>
</table>
| $2x^3 + 16x^2 + 6x + 24$ | **Step 1:** Check for C.F.: 2  
Factor it out. Don’t forget to carry it into the answer (see arrow) | **Remember:** Checking for a Common Factor is a possibility in every problem.  
**Note:** Once pulled out it is very easy to accidentally discard it. Draw an arrow to help you remember to bring it down into the answer.  
**Note:** Factoring by grouping doesn’t always work with 4 terms. We will not be factoring these. |
| $\frac{2x^3}{2} + \frac{16x^2}{2} + \frac{6x}{2} + \frac{24}{2}$ | **Step 2:** Break the 4 terms into a (binomial) + (binomial) | **You can check by multiplying.** |
| $2(x^3 + 8x^2 + 3x + 12)$ | **Step 3:** Pull out the C.F. from each binomial to get  
C.F. (binomial) ± C.F. (binomial) |  |
| $(x^3 + 8x^2) + (3x + 12)$ |  |
| $\frac{x^3 + 8x^2}{x^2} + \frac{3x + 12}{3}$ |  |
| $x^2(x + 8) + 3(x + 4)$ |  |
| $(x^3 + 8x^2) + (3x + 12)$ |  |
| $\frac{x^3 + 8x^2}{x^2} + \frac{3x + 12}{3}$ |  |
Perform the indicated operations.

1. \(3a(4b^2 - a)\)
2. \((x + 4)^2\)
3. \((x + 1)(x^2 - 3x - 4)\)
4. \((-x + 2)(3x - 4 + \frac{1}{-x^2})\)
5. \((-4m^3 - 16m^2 + 6m) \div (-2m)\)

Identify which method(s) you should try in factoring the following.

6. \(3x^2 + x\)
7. \(x^2 - 4x + 4\)
8. \(5c^3 + 10c^2 - 2c - 6\)
9. \(14x^3 + 7x^2 - 21x\)
10. \(a^2b - bc + 3bc^2 - c^2\)
11. \(-4t^2 - 5tr\)

Identify the greatest common factor between the terms.

12. \(18, 24, 48\)
13. \(3x, 9x^2, 6\)
14. \(-14s^3, -7s\)
15. \(a^2b, -bc, 3bc^2\)
16. \(125x^3, 50x^2, 10x\)
17. \(y^4, 4y^3, 2y^2, 8y\)

Factor the following by pulling out the greatest common factor if there is one. If not, the expression is prime. Check your answers.

18. \(9k + 3\)
19. \(-18y^5 - 6y\)
20. \(2x^4 - 3y^2 + 7y\)
21. \(-z^2 - 7a - 2\)
22. \(-12a^3b + 8a^2b^2 - 16ab^2\)
23. \(19xy^2 - 38xy + 57y\)
24. \(2a^2x^4 + 6a^2x^3 - 10ax^3\)
25. \(-39s^5 - 18s^3 - 81s\)

Factor the following by grouping if possible.

26. \(3x^3 - 9x^2 + 4x - 12\)
27. \(-2x^3 - 2x^2 - 3x - 3\)
28. \(4x^3 - 20x^2 - 6x + 10\)
29. \(x^3 + 5x^2 - 2x - 10\)
30. \(a^2x + 5a^2x + bx + 5b\)
31. \(8x^3 + 18x^2 - 20xy - 45y\)
32. \(x^3 + 3x^2 - 12x - 36\)
33. \(-4d^3 + 2d^2y - 6dy + 3y^2\)
34. \(x^3 + x^2 - x - 1\)
**Story Problem.**

35. This pasture has an area described by the polynomial \(10x^4 + 4x^3 - 15x^2 - 6x\). Its length and width are described by binomials. Find a solution set that will give the given area.

**Preparation.**

36. Multiply the following.
\[(x + 7)(x + 5)\quad (x + 2)(x + 8)\quad (x + 9)(x + 3)\quad (x + 5)(x + 4)\]

37. Given binomials like those in #36, notice that all of your answers simplified to trinomials. Describe how you get the middle term of those trinomials.
Answers
1. \(12ab^2 - 3a^2\)
2. \(x^2 + 8x + 16\)
3. \(x^3 - 2x^2 - 7x - 4\)
4. \(-3x^2 + 10x - 7\)
5. \(2m^2 + 8m - 3\)
6. Method 1: **Pull out Common Factors**
   Method 5: **Special Cases**
7. Method 1: **Pull out Common Factors**
   Method 3: \(ax^2 + bx + c, \text{where } a = 1\)
8. Method 1: **Pull out Common Factors**
   Method 2: **Grouping**
9. Method 1: **Pull out Common Factors**
   Method 4: \(ax^2 + bx + c, \text{where } a \neq 1\)
10. Method 1: **Pull out Common Factors**
    Method 2: **Grouping**
11. Method 1: **Pull out Common Factors**
    Method 5: **Special Cases**
12. 6
13. 3
14. \(-7s\)
15. \(b\)
16. \(5x\)
17. \(y\)
18. \(3(3k + 1)\)
19. \(-6y(3y^4 + 1)\)
20. **prime**
21. \(-1(z^2 + 7a + 2)\)
22. \(-4ab(3a^2 - 2ab + 4b)\)
23. \(19y(xy - 2x + 3)\)
24. \(2ax^3(ax + 3a - 5)\)
25. \(-3s(13s^4 + 6s^2 + 27)\)
26. \((x - 3)(3x^2 + 4)\)
27. \((x + 1)(-2x^2 - 3)\)
28. \(2(2x^3 - 10x^2 - 3x + 5)\)
   Not factorable by grouping
29. \((x^2 - 2)(x + 5)\)
30. \((a^2 + b)(x + 5)\)
31. Not factorable by grouping
32. \((x^2 - 12)(x + 3)\)
33. \((-2d + y)(2d^2 + 3y)\)
34. \((x + 1)(x^2 - 1)\)
35. side 1: \((2x^3 - 3x)\)
   side 2: \((5x + 2)\)
36. In class.
37. In class.
5.2 Factoring: Method 3
Trinomials: \(ax^2 + bx + c\), where \(a = 1\)

**OBJECTIVES**
- Factor polynomials of the form \(ax^2 + bx + c\) where \(a = 1\)

A  THE FIVE METHODS OF FACTORING: Review

**THE 5 METHODS of FACTORING**

<table>
<thead>
<tr>
<th>Method</th>
<th>Type of Polynomial used on:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Pull Out the Common Factor (C.F.)</td>
<td>All</td>
</tr>
<tr>
<td>2: Grouping</td>
<td>4 Terms</td>
</tr>
<tr>
<td>3: (ax^2 + bx + c), where (a = 1)</td>
<td>Trinomials, where (a = 1)</td>
</tr>
<tr>
<td>4: (ax^2 + bx + c), where (a \neq 1)</td>
<td>Trinomials, where (a \neq 1)</td>
</tr>
<tr>
<td>5: Special Cases</td>
<td>Binomials &amp; Trinomials</td>
</tr>
</tbody>
</table>

B  METHOD 3: \(x^2 + bx + c\), where \(a = 1\)

As you can see in the table, there are two methods of factoring when the polynomial is a
trinomial of the form \(ax^2 + bx + c\). The letters \(a\), \(b\), and \(c\) represent the coefficients of each term. For example, in the polynomial \(x^2 - 3x + 2\), we would say \(a = 1, b = -3,\) and \(c = 2\).

**Factoring Method 3:**
\[ax^2 + bx + c\ \text{where} \ a = 1\]

1. Check for method 1 first (Pull out C.F.)
2. Place the trinomial in descending order or \(x^2 + bx + c\).
3. Find factor pairs of \(c\).
4. Find the two factors of \(c\) that add up to \(b\).
5. Answer format: \((x \pm \text{factor})(x \pm \text{factor})\) The \(x\) is the square root of the first term. The two factors are from step 3, and their accompanying signs determine the \(\pm\).
**Step 2:** Find factor pairs of \( c \).

**Step 3:** Find the two factors of \( c \) that add up to \( b \).

**Step 4:** Answer format: \((x \pm factor)(x \pm factor)\) where \( x \) is the square root of the first term, and the factors come from Step 3.

---

**Factor: \( x^2 - 5x + 6 \)**

\[
x^2 - 5x + 6
\]

**Factors of 6** | **Sum of Factor Pairs**
---|---
1 × 6 | 1 + 6 = 7
2 × 3 | 2 + 3 = 5
(−1) × (−6) | (−1) + (−6) = −7
(−2) × (−3) | (−2) + (−3) = −5

\[
\sqrt{\text{first term}} = \sqrt{x^2} = x, \text{ factors are } -2 \text{ and } -3
\]

\[
(x \pm factor)(x \pm factor)
\]

\[
(x - 2)(x - 3)
\]

\[
\begin{align*}
(x)(x) + (x)(-3) + (-2)(x) + (-2)(-3) \\
x^2 - 3x - 2x + 6 \\
x^2 - 5x + 6
\end{align*}
\]

**Answer:** \((x - 2)(x - 3)\)

---

**Factor: \( 9 + m^2 + 6m \)**

\[
9 + m^2 + 6m
\]

\[
m^2 + 6m + 9
\]

**Factors of 9** | **Sum of Factor Pairs**
---|---
1 × 9 | 1 + 9 = 10
3 × 3 | 3 + 3 = 6
(−1) × (−9) | (−1) + (−9) = −10
(−3) × (−3) | (−3) + (−3) = −6

\[
\sqrt{\text{first term}} = \sqrt{m^2} = m, \text{ factors are } 3 \text{ and } 3
\]

\[
(x \pm factor)(x \pm factor)
\]

\[
(m + 3)(m + 3)
\]

\[
\begin{align*}
(m)(m) + (m)(3) + (3)(m) + (3)(3) \\
m^2 + 3m + 3m + 9 \\
m^2 + 6m + 9
\end{align*}
\]

**Answer:** \((m + 3)(m + 3)\) or \((m + 3)^2\)

---

**Important Points:**

- It is critical to **take the signs of the factors** found in step 3 into the answer to determine the ± signs.
- Always make sure the trinomial is in **descending order** so you can see clearly what \( b \) and \( c \) are.
- Does \((x - 3)(x - 2) = (x - 2)(x - 3)\)? **YES.** Multiply them out to prove it. Hence, it makes no difference in which order you place the two binomials.
3  **Factor: $2x^2 - 6x - 20$**

\[
2x^2 - 6x - 20
\]

C.F. = 2

\[
\begin{align*}
\frac{2x^2}{2} - \frac{6x}{2} - \frac{20}{2} \\
2(x^2 - 3x - 10)
\end{align*}
\]

<table>
<thead>
<tr>
<th>Factors of $-10$</th>
<th>Sum of Factor Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 \times (-1)$</td>
<td>$10 + (-1) = 9$</td>
</tr>
<tr>
<td>$(-10) \times 1$</td>
<td>$(-10) + 1 = -9$</td>
</tr>
<tr>
<td>$(-2) \times 5$</td>
<td>$(-2) + 5 = 3$</td>
</tr>
<tr>
<td>$(5) \times (-2)$</td>
<td>$(5) + (-2) = -3$</td>
</tr>
</tbody>
</table>

\[
\sqrt{\text{first term}} = \sqrt{2x^2} = x, \text{factors are } -5 \text{ and } 2
\]

\[
(x \pm \text{factor})(x \pm \text{factor})
\]

\[
(x - 5)(x + 2)
\]

**BUT don’t forget the common factor from the beginning:**

\[
2(x - 5)(x + 2)
\]

**Answer:** $2(x - 5)(x + 2)$

You can check by multiplying.

---

4  **Factor: $x^4 - 7x^2 + 12$**

\[
x^4 - 7x^2 + 12
\]

Step 1: Place the trinomial in descending order or $x^2 + bx + c$ - - already done

<table>
<thead>
<tr>
<th>Factors of 12</th>
<th>Sum of Factor Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 4$</td>
<td>$3 + 4 = 7$</td>
</tr>
<tr>
<td>$(-3) \times (-4)$</td>
<td>$(-3) + (-4) = -7$</td>
</tr>
</tbody>
</table>

\[
\sqrt{\text{first term}} = \sqrt{x^4} = x^2, \text{factors are } -3 \text{ and } -4
\]

\[
(x \pm \text{factor})(x \pm \text{factor})
\]

\[
(x^2 - 3)(x^2 - 4)
\]

**Answer:** $(x^2 - 3)(x^2 - 4)$

You can check by multiplying.

---

**Important Points:**
- The first term in $x^2 + bx + c$ form is not always an $x^2$. In example 4, it’s $x^4$.
- The first term in each ( ) in the answer must, when multiplied, = the first term in the original trinomial.
- It is not necessary to write all the factors of $c$. It is only requisite to find the two when multiplied = $c$, yet when added = $b$. Note in example 4, we did not list all the factors of $c$. 

Section 5.2
5.2 Exercise Set

Identify the greatest common factor between the terms.

1. 24, 96, 336
2. 120, 480, 960
3. 77, 154, 968
4. \(-13b^2, -12ab, -25b^2\)
5. \(12a^2, 16a^2, 96a^2\)
6. \(14m^2n, 28mn^2, 77m^2\)

Factor the following by pulling out the greatest common factor if possible.

7. \(-3x^2 + 6x^2y - 27x^3\)
8. \(7xy^2 - 2xy + 4x^2y + 2x^2y^2\)
9. \(12p^4 + 56p^2 - 48s\)
10. \(3r^2 + 27r^3 - 33r^4\)

Factor the following by grouping if possible.

11. \(28x^3 - 12x^2 + 7x - 3\)
12. \(6x^4 + 3x^3 + 2x^2 + x\)
13. \(x^3 - 3x^2 + x - 3\)
14. \(16x^3 + 52x^2 - 52x - 169\)
15. \(2a^2x - 2a^2y + 3bx - 3by\)
16. \(2x^3 + 2x^2y - 3x - 3y\)
17. \(x^3 - 3x^2 - 9x + 27\)
18. \(x^7 + 4x^6 + x^4 + 4x^3\)

Factor the following using the \(ax^2 + bx + c\), where \(a = 1\) method.

19. \(x^2 + 2x - 24\)
20. \(x^2 + 11x + 18\)
21. \(x^2 - 8x + 15\)
22. \(x^2 - x - 20\)
23. \(x^2 + 16x + 63\)
24. \(x^2 - 4x - 60\)
25. \(3x^2 + 27x + 24\)
26. \(x^2 - 7x - 60\)
27. \(x^2 + 6x - 27\)
28. \(7x^2 - 7x - 14\)

Story problem.

29. This box’s volume is described by the polynomial \(4x^3 - 12x^2 - 72x\). Its height is described by a monomial and its length and width are described by binomials. Find a solution set using prime factors.

Preparation: The following are trinomials in the form \(ax^2 + bx + c\) where \(a \neq 1\).

30. Multiply the following polynomials together.

\[(3x + 5)(2x + 7)\] \[(5x - 1)(4x + 3)\] \[(8x + 1)(2x + 1)\]
Answers

1. 24
2. 120
3. 11
4. \(-b\)
5. \(4a^2\)
6. \(7m\)
7. \(-3x^2(1 - 2y + 9x)\)
8. \(xy(7y - 2 + 4x + 2xy)\)
9. \(4(3p^2 + 14p^2 - 12s)\)
10. \(3r^2(1 + 9r - 11r^2)\)
11. \((7x - 3)(4x^2 + 1)\)
12. \((3x^3 + x)(2x + 1)\)
13. \((x^2 + 1)(x - 3)\)
14. \((4x^2 - 13)(4x + 13)\)
15. \((2a^2 + 3b)(x - y)\)
16. \((2x^2 - 3)(x + y)\)
17. \((x^2 - 9)(x - 3)\)
18. \((x^6 + x^3)(x + 4)\)
19. \((x + 6)(x - 4)\)
20. \((x + 9)(x + 2)\)
21. \((x - 3)(x - 5)\)
22. \((x - 5)(x + 4)\)
23. \((x + 9)(x + 7)\)
24. \((x - 10)(x + 6)\)
25. \(3(x + 8)(x + 1)\)
26. \((x - 12)(x + 5)\)
27. \((x + 9)(x - 3)\)
28. \(7(x - 2)(x + 1)\)
29. \textit{height}: 4x  
   \textit{length}: (x + 3)  
   \textit{width}: (x - 6)
30. In class
5.3 Factoring: Method 4
Trinomials: $ax^2 + bx +c$, where $a \neq 1$

**OBJECTIVES**
- Factor polynomials of the form $ax^2 + bx + c$ where $a \neq 1$

**THE FIVE METHODS OF FACTORING: Review**

**THE 5 METHODS of FACTORING**

<table>
<thead>
<tr>
<th>Method</th>
<th>Type of Polynomial used on:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Pull Out the Common Factor (C.F.)</td>
<td>All</td>
</tr>
<tr>
<td>2: Grouping</td>
<td>4 Terms</td>
</tr>
<tr>
<td>3: $ax^2 + bx +c$, where $a = 1$</td>
<td>Trinomials, where $a = 1$</td>
</tr>
<tr>
<td>4: $ax^2 + bx +c$, where $a \neq 1$</td>
<td>Trinomials, where $a \neq 1$</td>
</tr>
<tr>
<td>5: Special Cases</td>
<td>Binomials &amp; Trinomials</td>
</tr>
</tbody>
</table>

**METHOD 4: $ax^2 + bx +c$, where $a \neq 1$**

Always check for a common factor before you start trying other methods of factoring. This is especially important for trinomials of the form $ax^2 + bx + c$ when $a \neq 1$ because sometimes you can take out a common factor to reduce the trinomial to the form $ax^2 + bx + c$ when $a = 1$.

**Factoring Method 4:**

$ax^2 + bx +c$ where $a \neq 1$

1. Check for **method 1** first (Pull out C.F.)
2. Place the trinomial in descending order or $ax^2 + bx + c$.
3. Multiply $a \cdot c$.
4. Factor the product of $a \cdot c$ to find the two factors that add to $b$.
5. Break $bx$ into two terms equal to the two factors found in step 4.
6. You now have four terms. Factor by grouping.
**EXAMPLES**

1 **Factor: $2x^2 - x - 3$**

\[
\begin{align*}
2x^2 - x - 3 & \\
2x + 3 & = -1 \\
6 & = -6 \\
\end{align*}
\]

Step 1: Check for a common factor (in this case, none)
Step 2: Place the trinomial in descending order - already done
Step 3: Multiply $a \cdot c$.
Step 4: Factor the product of $a \cdot c$ to find the two factors that add to $b$. (We don’t need all the factors of $a \cdot c$, just the one that works)
Step 5: Break $bx$ into two terms equal to the two factors found in step 4.
Step 6: Factor the four terms by grouping.

**Answer:** $(2x - 3)(x + 1)$

You can check by multiplying.

**Important Point:**
You might wonder what would happen if you wrote the two terms from Step 5 in a different order. For example, from Example 1, would the answer be the same if we wrote $2x^3 + 2x - 3x - 3$ instead of $2x^3 - 3x + 2x - 3$? Try it and see. You will find that you get the same answer, just written in a different order.

This leads to the question, is $(2x - 3)(x + 1)$ the same as $(x + 1)(2x - 3)$? Think back to Chapter 1.7 and the commutative property of multiplication – you can multiply the two polynomials in any order. If you don’t believe it, try multiplying them out and see!

2 **Factor: $6y^2 + 25y + 25$**

\[
\begin{align*}
6y^2 + 25y + 25 & \\
6y + 15 & = 25 \\
150 & = 150 \\
\end{align*}
\]

Step 1: Check for a common factor (in this case, none)
Step 2: Place the trinomial in descending order - already done
Step 3: Multiply $a \cdot c$.
Step 4: Factor the product of $a \cdot c$ to find the two factors that add to $b$.
Step 5: Break $bx$ into two terms equal to the two factors found in step 4.
Step 6: Factor the four terms by grouping.

**Answer:** $(2y + 5)(3y + 5)$

You can check by multiplying.
### 3 Factor: $12x^2 - 13xy - 4y^2$

$$12x^2 - 13xy - 4y^2$$  
$$a = 12, c = -4$$  
$$b = -13$$  
$$a \cdot c = (12) \cdot (-4) = -48$$  

**Step 1:** Check for a common factor (in this case, none)  
**Step 2:** Place the trinomial in descending order - already done  
**Step 3:** Multiply $a \cdot c$.  

**Factors of $-48$**  
- $-16 \times 3$  
- $-16 + 3 = -13$  

**Sum of Factor Pairs**  
- $-16 + 3 = -13$  

$$12x^2 - 13xy - 4y^2$$  
$$12x^2 - 16xy + 3xy - 4y^2$$  
$$4x(3x - 4y) + y(3x - 4y)$$  
$$(3x - 4y)(4x + y)$$  

**Answer:** $(3x - 4y)(4x + y)$  
You can check by multiplying.

### 4 Factor: $-16x + 4 + 16x^2$

$$-16x + 4 + 16x^2$$  
$$= 4(-4x + 1 + 4x^2)$$  
$$4(4x^2 - 4x + 1)$$  
$$a = 4, c = 1$$  
$$b = -4$$  
$$a \cdot c = (4) \cdot (1) = 4$$  

**Step 1:** Check for a common factor (in this case, $4$)  
**Step 2:** Place the trinomial in descending order  
**Step 3:** Multiply $a \cdot c$.  

**Factors of $4$**  
- $(-2) \times (-2)$  
- $(-2) + (-2) = -4$  

**Sum of Factor Pairs**  
- $-2 + (-2) = -4$  

$$4(4x^2 - 4x + 1)$$  
$$4[4x^2 - 2x - 2x + 1]$$  
$$4[2x(2x - 1) - 1(2x - 1)]$$  
$$4(2x - 1)(2x - 1)$$  

**Answer:** $4(2x - 1)(2x - 1)$ or $4(2x - 1)^2$  
You can check by multiplying.
**Factor: 8x^2 − 12x − 60**

\[
8x^2 - 12x - 60 = 4(2x^2 - 3x - 15)
\]

- \(a = 2, c = -15\)
- \(b = -3\)
- \(a \cdot c = (2) \cdot (15) = 30\)

**Step 1:** Check for a common factor (in this case, 4)

**Step 2:** Place the trinomial in descending order

**Step 3:** Multiply \(a \cdot c\).

**Factors of 4** | **Sum of Factor Pairs**
---|---
-30 \(\times\) 1 | -30 + 1 = -29
-15 \(\times\) 2 | -15 + 2 = -13
-10 \(\times\) 3 | -10 + 3 = -7
-6 \(\times\) 5 | -6 + 5 = -1

4(2x^2 − 3x − 15)

**Step 4:** Factor the product of \(a \cdot c\) to find the two factors that add to \(b\).

No factor pairs add to -3, so factoring is impossible by **method 4**. However, the polynomial is not **prime** because there was a C.F.

**Answer:** 4(2x^2 − 3x − 15)

You can check by multiplying.
5.3 EXERCISE SET

Identify the greatest common factor between the terms.

5.1 1. 14, 49, 112  
       2. 15x, 39xy, 52x  
       3. $4j^2k, -80j^2k^3, 105k^2$

Factor the following by pulling out the greatest common factor.

4. $49x^5 + 21x^3 - 14x^2$  
6. $15x^2y^3z^2 - 12x^2y^3z + 9x^2y^3$  
5. $24x^3y^3 + 96x^2y^3 - 72x^3y^2$

Factor the following by grouping if possible.

8. $8x^3 + 2x^2 - 12x - 15$  
10. $2x^3 + 3x^2 + 2x + 3$  
9. $3x^3 - 15x^2 + 5x - 25$

Factor the following using the $ax^2 + bx + c$, where $a = 1$ method.

12. $x^2 - 5x - 84$  
14. $x^2 - 2x - 35$  
13. $x^2 - x - 6$  
15. $x^2 - 15x + 54$

Factor the following using the $ax^2 + bx + c$, where $a \neq 1$ method.

18. $10x^2 - 7x - 6$  
19. $-8x^2 - 2x + 3$  
20. $3 + 16x + 5x^2$  
21. $-2m^2 - 32m - 12$

Preparation: Apply the rules learned in Section 4.3 about Special Cases of Multiplication for Binomials to answer the following problems.

4.3 28. $(x - 5)(x + 5)$  
29. $(ax + b)^2$
Answers
1. 7
2. x
3. 5k
4. $7x^2(7x^3 + 3x - 2)$
5. $24x^2y^2(xy + 4y - 3x)$
6. $3x^2y^3(5z^2 - 4z + 3)$
7. $2(9p^3 - 3p^2 + 7p^4 + rs)$
8. Not factorable by Grouping
9. $(3x^2 + 5)(x - 5)$
10. $(x^2 + 1)(2x + 3)$
11. $(x^2 - 1)(x + 1)$
12. $(x - 12)(x + 7)$
13. $(x - 3)(x + 2)$
14. $(x - 7)(x + 5)$
15. $(x - 9)(x - 6)$
16. $(x - 9)(x - 9)$
17. $(x - 11)(x + 3)$
18. $(5x - 6)(2x + 1)$
19. $(2x - 1)(-4x - 3)$
20. $(5x + 1)(x + 3)$
21. $-2(m^2 + 16m + 6)$
   C.F. but not factorable by $ax^2 + bx + c$, where $a \neq 1$
22. $(2x + 5)(-x + 5)$
23. $(3x + 2y)(2x + 7y)$
24. $(-2s + 5)(s - 8)$
25. $2(4x - 1)(2x + 3)$
26. $(5x + y)(x - 3y)$
27. $(-y + 4)(7y - 2)$
28. In class.
29. In class.
**SQUARE ROOTS ($\sqrt{\_}$):** They look scary, but simply are shortcuts asking the question, “What number or expression times itself equals the number or expression under the radical sign,$\sqrt{\_}$?”

**Examples:**

- $\sqrt{4} = 2$ because $2 \times 2 = 4$
- $\sqrt{x^2} = x$ because $x \cdot x = x^2$

**PERFECT SQUARE ROOTS:** An integer or variable multiplied times itself equals the expression under the $\sqrt{\_}$.

**Examples:**

- $\sqrt{16} = 4$, $\sqrt{100} = 10$, $\sqrt{x^2} = x$,
- $\sqrt{p^2w^2} = pw$

Each of these perfect square roots can be reduced to an integer or variable (or a combination of the two).

**NON-PERFECT SQUARE ROOTS:** No integer or variable multiplied times itself equals the expression under the $\sqrt{\_}$.

**Examples:**

- $\sqrt{13}, \sqrt{85}, \sqrt{y}, \sqrt{x^3}$

None of these square roots can be reduced down to an integer or variable. Nothing times itself will equal the value under the radical sign.
DIFFERENCE OF TWO SQUARES: This means just what it says: a difference (one number subtracting the other) of two squares (two terms whose coefficients and variables are perfect squares).

Examples: \(x^2 - 16\) \(4y^2 - 9\) \(36 - m^2n^2\) \(25x^4 - 1\)

Can you see how each of these terms is a perfect square? Another important thing to note: each term has a different sign. As long as only one of the numbers is negative, it will work. For example, \(-16 + x^2\) can be rearranged to be \(x^2 - 16\) by the commutative property of addition.

Factoring Method 5: Special Cases

Difference of Two Squares

\((A^2 - B^2) = (A + B)(A - B)\)

1. Check for method 1 first (Pull out C.F.)
   CHECK: Is this a binomial of the form \((A^2 - B^2)\)? Are both terms perfect squares?
2. Take the square root of each term (don’t include signs)
3. Answer format: \((A + B)(A - B)\)
   or \((\sqrt{1^{st} \text{ term}} + \sqrt{2^{nd} \text{ term}})(\sqrt{1^{st} \text{ term}} - \sqrt{2^{nd} \text{ term}})\)
4. Repeat these steps until no factoring is possible

While a difference of two squares is a wonderful shortcut, do not make the mistake of using the same shortcut on a sum of two squares. The sum of two squares is always prime if there is no common factor.

EXAMPLES

1. Factor: \(3p^2 - 363\)

\[3p^2 - 363 = 3(p^2 - 121)\]  Step 1: Check for a common factor (in this case, 3)
\[\sqrt{p^2} = p; \ \sqrt{121} = 11\]  Step 2: Take the square root of each term
\[3(p + 11)(p - 11)\]  Step 3: Answer format: \((A + B)(A - B)\)
No more factoring possible.  Step 4: Repeat these steps until no factoring is possible

Answer: \(3(p + 11)(p - 11)\)
You can check by multiplying.
Factor: \( 49x^2 - 16y^2 \)

\[
49x^2 - 16y^2
\]

Step 1: Check for a common factor (in this case none)

\[
\sqrt{49x^2} = 7x ; \quad \sqrt{16y^2} = 4y
\]

Step 2: Take the square root of each term

\[
(7x + 4y)(7x - 4y)
\]

Step 3: Answer format: \((A + B)(A - B)\)

Step 4: Repeat these steps until no factoring is possible

Answer: \((7x + 4y)(7x - 4y)\)

You can check by multiplying.

Factor: \( y^4 - 36 \)

\[
y^4 - 36
\]

Step 1: Check for a common factor (in this case none)

\[
\sqrt{y^4} = y^2 ; \quad \sqrt{36} = 6
\]

Step 2: Take the square root of each term

\[
(y^2 + 6)(y^2 - 6)
\]

Step 3: Answer format: \((A + B)(A - B)\)

Step 4: Repeat these steps until no factoring is possible

Answer: \((y^2 + 6)(y^2 - 6)\)

You can check by multiplying.

You may run into some problems that aren’t quite what they seem. See the following examples.

Factor: \(-x^2 + 16\)

\[
-x^2 + 16
\]

At first glance, this looks like a sum of squares which would be prime. However, we know that by the commutative property of addition that we can rearrange this binomial. Now you can see it is a difference of squares and can be factored as normal.

\[
\sqrt{16} = 4 ; \quad \sqrt{x^2} = x
\]

\[
(4 + x)(4 - x)
\]

Answer: \((4 + x)(4 - x)\)

You can check by multiplying.

Factor: \(9x^2 - 18\)

\[
9x^2 - 18
\]

Step 1: Check for a common factor (in this case, 9)

\[
\sqrt{9(x^2 - 2)}
\]

Step 2: Take the square root of each term

--This is when we realize that this binomial is not a perfect square. Therefore, no more factoring is possible.

Answer: \(9(x^2 - 2)\)
Factor: $y^4 - 1$

\[
y^4 - 1 = \sqrt{y^4} = y^2; \quad \sqrt{1} = 1
\]

\[
(y^2 + 1)(y^2 - 1)
\]

Prime $\sqrt{y^2} = y; \quad \sqrt{1} = 1$

\[
(y^2 + 1) (y + 1)(y - 1)
\]

Answer: $(y^2 + 1)(y + 1)(y - 1)$

**COMMON QUESTION: WHY IS THE SUM OF SQUARES PRIME?**

This question is best answered by an experiment in checking our answers. Remember, in one sense, factoring is the opposite of multiplication. So if there’s nothing that when multiplied equals the sum of two squares, it’s going to be prime.

**Example:** $x^2 + 4$ – We must have in our (binomial)×(binomial) answer an $(x \, \Box \, 2)(x \, \Box \, 2)$ to make the $x^2$ and 4 in $x^2 + 4$. The question then is what signs go in between them replacing the $\Box$’s to make it work? There are only three options:

1. $(x + 2)(x + 2)$  
   \[
x(x) + x(2) + 2(x) + 2(2)
   \]
   $x^2 + 2x + 2x + 4$
   $x^2 + 4x + 4$

2. $(x - 2)(x - 2)$  
   \[
x(x) + x(-2) + (-2)(x) + (-2)(-2)
   \]
   $x^2 - 2x - 2x + 4$
   $x^2 - 4x + 4$

3. $(x + 2)(x - 2)$ same as $(x - 2)(x + 2)$  
   \[
x(x) + x(-2) + 2(x) + 2(-2)
   \]
   $x^2 - 2x + 2x + 4$
   $x^2 - 4$

The only conclusion is that the Sum of Squares is impossible to factor, because no two factors when multiplied can ever equal the sum of the squares.

**METHOD 5: SPECIAL CASES: Perfect Square Trinomials**

This method is a shortcut that only applies to some trinomials, not all. Factoring methods 3 & 4 (Sections 5.2 & 5.3) take care of all trinomials. This shortcut is worth taking a look at, but if you find it too difficult or too hard to remember, you can always fall back on methods 3 & 4.

**PERFECT SQUARE TRINOMIALS:** This is what results when a binomial is squared – a binomial times itself. You will remember from 4.3 the following formula: $(A \pm B)^2 = A^2 \pm 2AB + B^2$. It is this trinomial that is a perfect square trinomial, because it is a square of a binomial.

**Example:** $(3x - 4)^2 = 9x^2 - 24x + 16$

\[
A \quad B \quad A^2 \quad 2AB \quad B^2
\]
We’ve worked with squaring binomials to get the perfect square binomials, but not we will go backwards. We will start with a perfect square trinomial and factor it down to a binomial squared.

**Factoring Method 5: Special Cases**

**Perfect Square Trinomials**

\[(A \pm B)^2 = A^2 \pm 2AB + B^2\]

1. Check for **method 1** first (Pull out C.F.)
2. **CHECK:**
   - Is it a trinomial?
   - Are the first and last terms perfect squares?
   - Is the middle term \(2 \times \sqrt{\text{first term}} \times \sqrt{\text{second term}}\)?
   - Is the last term positive?
   - If the answer is YES to all of the above, continue. If not, try another method.
3. Take the square root of the first and last terms. \(\sqrt{\text{first term}} = A, \sqrt{\text{second term}} = B\)
4. Answer format: \((A \pm B)^2\) the sign is decided by the first sign in the trinomial
   or \((\sqrt{\text{first term}} \pm \sqrt{\text{second term}})^2\)

**EXAMPLES**

The hardest part is recognizing the perfect square trinomials. Once you know what it is, the rest is easy. This is an exercise in recognizing perfect square trinomials.

<table>
<thead>
<tr>
<th>POLYNOMIAL</th>
<th>Checklist to recognize a Perfect Square Trinomial.</th>
<th>Perfect Square Trinomials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tri-nominal</td>
<td>Ends = Perfect Squares</td>
</tr>
<tr>
<td>(16y^2 + 24y + 9)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(16y^2 + 25y + 9)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(4x^2 + 12x + 9)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(4x^2 + 12x – 9)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(x^2 – 20x + 100)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(x^2 + 20x – 100)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(x^2 + 25x + 100)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>(x^2 + 20x + 96)</td>
<td>✔️</td>
<td>No</td>
</tr>
</tbody>
</table>
7. Factor: $16y^2 + 24y + 9$

$16y^2 + 24y + 9$

Step 1: Check for a common factor (in this case, none)

Step 2: Check the criteria for perfect square trinomials

$\sqrt{16y^2} = 4y = A$; $\sqrt{9} = 3 = B$

$(4y + 3)^2$

Step 3: Take the square root of the first and last terms ($A$ & $B$)

Step 4: Answer format: $(A \pm B)^2$

Answer: $(4y + 3)^2$

8. Factor: $8x^2 - 24x + 18$

$8x^2 - 24x + 18$

Step 1: Check for a common factor (in this case, 2)

Step 2: Check the criteria for perfect square trinomials

$\sqrt{4x^2} = 2x = A$; $\sqrt{9} = 3 = B$

$2(2x - 3)^2$

Step 3: Take the square root of the first and last terms ($A$ & $B$)

Step 4: Answer format: $(A \pm B)^2$. The answer will have a negative sign because first term in the polynomial in also negative.

Answer: $2(2x - 3)^2$

9. Factor: $25x^2 + 15x + 36$

$25x^2 + 15x + 36$

Step 1: Check for a common factor (in this case, none)

Step 2: Check the criteria for perfect square trinomials – the middle term does not qualify

$\sqrt{25x^2} = 5x = A$; $\sqrt{36} = 6 = B$

$2AB = 2(5x)(6) = 60x \neq 15x$

Answer: Prime

10. Factor: $x^2 - 20x + 100$

$x^2 - 20x + 100$

Step 1: Check for a common factor (in this case, 2)

Step 2: Check the criteria for perfect square trinomials

$\sqrt{x^2} = x = A$; $\sqrt{100} = 10 = B$

$(x - 10)^2$

Step 3: Take the square root of the first and last terms ($A$ & $B$)

Step 4: Answer format: $(A \pm B)^2$. The answer will have a negative sign because first term in the polynomial in also negative.

Answer: $(x - 10)^2$
Important Notes:

- Perfect square trinomials can make factoring faster if you can recognize them
- Perfect square trinomials can always be factored using methods 3 or 4 instead.

  o Example: Let’s factor the polynomial in Example 7 using method 4, and see if we get the same answer.

Step 1: Check for a common factor (in this case, none)
Step 2: Place the trinomial in descending order - already done
Step 3: Multiply $a \cdot c$.
Step 4: Factor the product of $a \cdot c$ to find the two factors that add to $b$.
Step 5: Break $bx$ into two terms equal to the two factors found in step 4.
Step 6: Factor the four terms by grouping.

Now you can see how much faster it can be if you recognize the perfect square trinomial to begin with.
Factor the following by pulling out the greatest common factor if possible.

1. $8n^1g_1877^n - 48n^1g_1877^n + 64n^1g_1877^n$
2. $3j^2kb^3 - 2j^3k^5b^2 + 5j^3k^2b - 7j^3k^3b^3$

Factor the following by grouping if possible.

3. $3n^1g_1872^n - 2n^1g_1873^n - 2n^1g_1854^n$
4. $5n^1g_1872^n - 7n^1g_1872^n - 63$

Factor the following using the $ax^2 + bx + c$, where $a = 1$ method.

5. $x^2 - 2x - 63$
6. $x^2 + 20x + 75$

Factor the following using the $ax^2 + bx + c$, where $a \neq 1$ method.

7. $12x^2 + 7x - 10$
8. $-14x^2 + 17x + 6$

Story problem.

9. An alien spaceship has traveled $10x^2 + 19x - 15$ miles from their home planet. Their speed and time can both be represented by binomials. Find two suitable binomials that will represent them. (Recall that $d = rt$, and you have been given distance.)

Determine if the following are differences of squares, then factor. If unfactorable, explain why.

10. $x^2 - 36$
11. $16y^4 + 9$
12. $4x^2 - 9$
13. $54x^2 - 24$
14. $-25 + 4y^2$
15. $25g^8 - 81$
16. $27 - 3m^2$
17. $y^5 + 4y$
18. $2x^2 - 1$
19. $16x^2 - 49$

Determine if the following are perfect square trinomials. If they are, factor using method 5. If they are not or you are unsure, use method 3 or 4.

20. $x^2 + 10x + 25$
21. $4x^2 - 12x + 9$
22. $3x^2 + 5x - 2$
23. $6x^2 - 84x + 294$
24. $9y^4 - 66y^2 + 121$
25. $2m^2 + 16m + 32$
26. $4x^2 - 16x + 16$
27. $4x^2 + 30x - 100$

Preparation: The following polynomials have been factored already. Determine if they are completely factored. If not, finish factoring the polynomials.

28. $4(x^2 - 6x + 9)$
29. $(x^2 + 4)(x^2 - 4)$
30. $3(m - 8)^2$
31. $7(100y^4 - 16)$
Answers

1. \(8x^3(x^3 + 8x - 6)\)
2. \(j^2kb(3b^2 - 2jk^4b + 5jk - 7jk^2b^2)\)
3. \((3kb + 2)(4k^2 + 5)\)
4. \((7m^2n^2 - 1)(12m + 5)\)
5. \((x - 9)(x + 7)\)
6. \((x + 5)(x + 15)\)
7. \((3x - 2)(4x + 5)\)
8. \((-2x + 3)(7x + 2)\)
9. rate: \((2x + 5)\)  time: \((5x - 3)\) or rate: \((5x - 3)\)  time: \((2x + 5)\)
10. \((x + 6)(x - 6)\)
11. Prime – sum of squares
12. \((2x + 3)(2x - 3)\)
13. \(6(3x + 2)(3x - 2)\)
14. \((2y + 5)(2y - 5)\)
15. \((5g^4 + 9)(5g^4 - 9)\)
16. \(3(3 + m)(3 - m)\)
17. \(y(y^4 + 4)\)
18. Prime – \(2\) is not a perfect square
19. \((4x + 7)(4x - 7)\)
20. \((x + 5)^2\)
21. \((2x - 3)^2\)
22. \((x + 2)(3x - 1)\)
23. \(6(x - 7)^2\)
24. \((3y^2 - 11)^2\)
25. \(2(m + 4)^2\)
26. \(4(x - 2)^2\)
27. \(2(2x - 5)(x + 10)\)
28. In class.
29. In class.
30. In class.
31. In class.
We have learned several methods of factoring, and each method is used in different circumstances. If you are unsure what to do when factoring a polynomial, this chart will be helpful.

**Steps to Factoring Polynomials**

1. **Always** check for a common factor.
2. Then look at the number of terms.
3. **2 terms**
   - Is the binomial a difference of two squares?
     - No: Prime

4. **3 terms**
   - Is it a perfect square trinomial?
     - Yes: \( A^2 + 2AB + B^2 = (A + B)^2 \)
     - No: Use the \( ac \) method. Multiply \( a \) and \( c \) and find factors that add to \( b \). Split the middle term into two, then factor by grouping.

5. **4 terms**
   - Factor by grouping

You can ALWAYS check your answer by multiplying.
Factor: $36y^2 - 36y + 9$

Check for a common factor.  
- $9(4y^2 - 4y + 1)$

How many terms?  
- Three terms

Is it a perfect square trinomial?  
- Yes, it follows the form $A^2 + 2AB + B^2$ where $A = 2y$ and $B = -1$

Factor knowing that $A^2 + 2AB + B^2 = (A + B)^2$.  
- $9(2y - 1)^2$

Check – can any of the factors be factored?  
- The factors are $9$ and $(2y - 1)$. Both are prime.

Check by multiplying.  
- ✓

Answer: $9(2y - 1)^2$

Factor: $81 - x^4$

Check for a common factor.  
- None

How many terms?  
- Two terms

Is it a difference of two squares?  
- Yes, $81$ and $y^4$ are both perfect squares

Factor knowing that $A^2 - B^2 = (A + B)(A - B)$.  
- $(9 + y^2)(9 - y^2)$

Check – can any of the factors be factored?  
- Yes, $(9 - y^2)$ can be factored

How many terms?  
- Two terms

Is it a difference of two squares?  
- Yes, $9$ and $y^2$ are both perfect squares

Factor  
- $(3 + y)(3 - y)$

What are all the factors of the original polynomial?  
- $(9 + y^2)(3 + y)(3 - y)$

Check – can any of the factors be factored?  
- No, all factors are now prime.

Check by multiplying.  
- ✓

Answer: $(9 + y^2)(3 + y)(3 - y)$

Factor: $x^2 + 18x + 77$

Check for a common factor.  
- No common factors

How many terms?  
- Three terms

Is it a perfect square trinomial?  
- Not a perfect square trinomial

What is the first coefficient? ($a = 1$ or $a \neq 1$)  
- $a = 1$, polynomial looks like $x^2 + bx + c$.

Factor – find factors of $c$ that add to $b$.  
- $c = 77 = 11 \times 7$; $b = 18 = 11 + 7$

Factored form: $(x + 11)(x + 7)$

Check – can any of the factors be factored?  
- No, all factors are now prime.

Check by multiplying.  
- ✓

Answer: $(x + 11)(x + 7)$
Factor the following. If non-factorable label as prime.

1. $1 - y^2$
2. $2x^2 - 8x + 8$
3. $13x + 3 + 18x^2$
4. $81 + x^4$
5. $-2s^2 + 21s - 40$
6. $6x^2 + 25xy + 14y^2$
7. $-5x + 2x^4 + 2x^2 - 5x^3$
8. $x^2 - y^2$
9. $16b^2 - 9b$
10. $4x^2 + 30x - 100$
11. $28x^2 + 65x + 28$
12. $4a^5 + 16$
13. $x^3 + 3x^2 - 4x - 12$
14. $r^3 + r^2 - 4r - 4$
15. $2x^3y + 4x^2y - 30xy$
16. $16z^3 + 48z^2 + 36z + 108$
17. $4x^5 + 12x^4 - 4x^3 + 12x^2$
18. $3x^2 + 5x - 17$
19. $x^8 - 81$
20. $12x^3y - 27xy^3$
21. $x^2 - 3x - 18$
22. $y^6 - k^{12}$
23. $16a^2 + 40a + 25$
24. $z^4 + 8z^2 + 16$
25. $2\pi r^2 - 2\pi$
26. $9x^2 + 3x$
27. $4b^2 + 256$
28. $18x^3 + 54x^2 + 6x + 18$

Story Problems.

29. A billboard along the side of I-15 has an area represented by the polynomial $9y^8 - 100$. Find 2 binomials that represent the length and width of billboard. Recall that $Area = (length) \times (width)$.

30. A telephone booth with a square bottom has a volume of $1000x^3 + 800x^2 + 160x$. Its height is represented by a monomial and its length and width by binomials. Find a monomial and two binomials that will represent these three dimensions.

Preparation:

31. Solve the following equations for the variable:
   a. $x^2 = 4$
   b. $25 - y^2 = 0$
Answers
1. \((1 + y)(1 - y)\)
2. \(2(x - 2)^2\)
3. \((2x + 1)(9x + 2)\)
4. Prime
5. \((-2s + 5)(s - 8)\)
6. \((3x + 2y)(2x + 7y)\)
7. \(x(x^2 + 1)(2x - 5)\)
8. \((x + y)(x - y)\)
9. \(b(16b - 9)\)
10. \(2(2x - 5)(x + 10)\)
11. \((4x + 7)(7x + 4)\)
12. \(4(a^5 + 4)\)
13. \((x + 2)(x - 2)(x + 3)\)
14. \((r - 2)(r + 2)(r + 1)\)
15. \(2xy(x - 3)(x + 5)\)
16. \(4(4z^2 + 9)(z + 3)\)
17. \(4x^2(x^3 + 3x^2 - x + 3)\)
18. Prime
19. \((x^4 + 9)(x^2 + 3)(x^2 - 3)\)
20. \(3xy(2x - 3y)(2x + 3y)\)
21. \((x + 3)(x - 6)\)
22. \((y^4 + k^6)(y^2 + k^3)(y^2 - k^3)\)
23. \((4a + 5)^2\)
24. \((z^2 + 4)^2\)
25. \(2\pi(r + 1)(r - 1)\)
26. \(3x(3x + 1)\)
27. \(4(b^2 + 64)\)
28. \(6(3x^2 + 1)(x + 3)\)
29. \(side 1: 3y^4 + 10\)
   \(side 2: 3y^4 - 10\)
30. \(height: 40x\)
   \(base length: 5x + 2\)
   \(base width: 5x + 2\)
31. In class.
**5.6 Solving Equations by Factoring**

**OBJECTIVES**
- Learn the principle of zero products
- Use the principle of zero products to solve equations
- Solve real-life situations using factoring

**THE PRINCIPLE OF ZERO PRODUCTS**

**DEFINITIONS & BASICS**

**PRINCIPLE OF ZERO PRODUCTS:** If \( a \cdot b = 0 \), then either \( a \) or \( b \) or both are equal to zero.

Consider the equation used to define the above principle: \( a \cdot b = 0 \)
What do we know about the variables? We know that at least one of those variables must equal zero. For example, if \( a \) is zero, then it doesn’t matter what \( b \) is; the equation will still be true. The same is true if \( b \) is zero, or both \( a \) and \( b \) are zero. This is the logic behind the principle of zero products.

We can also apply this principle to larger factors being multiplied together.

\[(x + 3)(x - 3) = 0\]

This means that either \( x + 3 \) or \( x - 3 \) equals 0 or they both equal 0. Please note that \( x \) could have up to 2 answers. In order to solve this we will split them up into separate equations and solve for \( x \) separately.

\[
\begin{align*}
(x + 3)(x - 3) &= 0 \\
x + 3 &= 0 \quad x - 3 &= 0 \\
-3 &= -3 \quad +3 &= +3 \\
x &= -3 \quad x &= 3
\end{align*}
\]

Therefore, we can say that \( x = -3 \) or 3 which we generally write in the form \( x = -3, 3 \).

This is all done by the principle of zero products. In order to use this principle, the equation must be equal to zero.

**Solving Equations by Factoring**

1. Arrange the equation so that it is equal to zero.
2. Factor the polynomial.
3. Set each factor equal to zero.
4. Solve each new equation for the variable.
5. Check your answer by substituting your answers into the original equation.

Section 5.6
**EXAMPLES**

1. **Factor: \(-5 - 7x = 2x^2\)**

   \[
   2x^2 + 7x + 5 = 0 \\
   (2x + 5)(x + 1) = 0 \\
   2x + 5 = 0 \quad x + 1 = 0
   \]

   Step 1: Arrange equation so that it equals zero.  
   Step 2: Factor the polynomial.  
   Step 3: Set each factor equal to zero.  
   Step 4: Solve each new equation for the variable.  
   Step 5: Check you answer by substituting your answers into the original equation.

   \[
   2x + 5 = 0 \\
   2x = -5 \\
   x = -\frac{5}{2}
   \]

   Check \(x = -\frac{5}{2}\)

   \[
   -5 - 7\left(\frac{-5}{2}\right) = 2\left(\frac{-5}{2}\right)^2 \\
   -5 + \frac{35}{2} = 2\left(\frac{25}{4}\right) \\
   \frac{25}{2} = \frac{25}{2} \checkmark
   \]

   Check \(x = -1\)

   \[
   -5 - 7(-1) = 2(-1)^2 \\
   -5 + 7 = 2 \\
   2 = 2 \checkmark
   \]

   Answer: \(x = -3, 2\)

2. **Factor: \(x^2 - 8x = -16\)**

   \[
   x^2 - 8x + 16 = 0 \\
   (x - 4)(x - 4) = 0 \text{ or } (x - 4)^2 = 0
   \]

   Step 1: Arrange equation so that it equals zero.  
   Step 2: Factor the polynomial.  
   Step 3: Set each factor equal to zero.  
   Step 4: Solve each new equation for the variable.  
   Step 5: Check you answer by substituting your answers into the original equation.

   \[
   x - 4 = 0 \\
   x = 4 \\
   +4 + 4 \\
   x = 4 \checkmark
   \]

   Check \(x = 4\)

   \[
   (4)^2 - 8(4) = -16 \\
   16 - 32 = -16 \\
   -16 = -16 \checkmark
   \]

   Answer: \(x = 4\)

Section 5.6
3 \textbf{Factor: }3x^3 + 15x^2 + 18x = 0

\begin{align*}
3x^3 + 15x^2 + 18x &= 0 \\
3x(x + 2)(x + 3) &= 0 \\
3x &= 0 \quad x + 2 = 0 \quad x + 3 = 0
\end{align*}

\begin{align*}
x &= 0 \quad x &= -2 \quad x &= -3
\end{align*}

\text{Answer: } x = -3, -2, 0

You can check all three answers by substituting into the original equation.

\textbf{COMMON MISTAKES}

- A common pitfall is that students assume \( x \) to be equal to the other number in the factor.

\textbf{Example: }\( x^2 + 14x + 33 = 0 \)

\((x + 11)(x + 3) = 0 \)

\( x = -11 \text{ and } -3 \) \textbf{not} 3 and 11

- Another common mistake is forgetting to get the equation to equal zero.

\textbf{Example: }\( x^2 - 19x + 52 = -32 \)

\begin{align*}
\text{Right} & \quad x^2 + 3x - 10 = 18 \\
& -18 - 18 \\
& x^2 + 3x - 28 = 0 \\
& (x + 7)(x - 4) = 0 \\
\end{align*}

\begin{align*}
x + 7 &= 0 \quad x - 4 = 0 \\
-7 &= -7 \quad +4 + 4 \\
x &= -7 \quad x = 4 \\
\end{align*}

\( x = -7, 4 \)

\textbf{THE CONNECTION}

You may well be wondering what this has to do with anything in “real life.” Let’s start with a graph of a quadratic equation.
Section 5.6

All quadratics (equations in which the highest power is 2, like this one) are in the shape of a parabola. The equation $y = x^2 - 4$ describes the shape on this graph. Remember from chapter 3 that to solve for the x-intercepts, we replace the $y$ with a 0 to get $0 = x^2 - 4$. Now this is a familiar polynomial in an equation that we can solve. We can factor to $0 = (x + 2)(x - 2)$, and we find that $x = -2, 2$. You can see on the graph that this is exactly where the parabola intersects the y-axis. This is the reason quadratics have two answers while linear equations (of the form $y = mx + b$) only have one answer – because of the number of times the graph crosses the y-axis.

Here is a graph of a quadratic that only has one answer. Look at the equation that describes the graph: $y = x^2 + 6x + 9$. You’ll notice this is a perfect square trinomial. By substituting 0 for $y$ to find the x-intercepts, we get $0 = x^2 + 6x + 9$, which factors to $0 = (x + 3)^2$. This means that $x = -3$, and that is the only answer because it is the only x-intercept.

You’re still thinking, “What does this have to do with anything?” Here is an example from physics:

Wiley Coyote ran off a cliff. His motion is described by the equation $d = -5t^2 + 5t + 10$, where $d$ is the distance he travels in meters and $t$ is the time in seconds that he travels. The question is, how long does it take him to hit the ground? In others words, what is time $t$ when the distance is 0. We can substitute 0 in for $d$ and factor the equation. We get $0 = (-5t + 10)(t + 1)$, which means $t = -1, 2$. Before we say that this is our final answer, let’s remember what $t$ really stands for: time. Since we can’t have a negative time, then $t = 2$ is the only answer. What does this mean? It means it takes Wiley Coyote 2 seconds to fall from the cliff. On the right is the graph of this situation. The left side of the graph is shaded because usually it wouldn’t be included in the graph because it is the area of “negative time.” However, now you can see that if negative time existed, then $x = -1$ would be a valid answer.
Now let’s look at some non-graphical examples. Recall from chapter 2 the process of solving story problems:

**D - Data.** Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

**V - Variable.** In all of these story problems, there is something that you don’t know, that you would like to. Pick any letter of the alphabet to represent this.

**P - Plan.** Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. Find a formula or draw a picture that helps you describe what is happening.

**E - Equation.** Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

*In quadratic equations, you usually get two answers. Only choose the answers that make logical sense, i.e. don’t choose a negative distance or a negative time.*

---

**EXAMPLES**

Jim is building a box for his rock collection. When all of his rocks are laid out, they take up an area of 96 square inches. He wants the length of the bottom of the box to be 4 inches longer than the width. What should the length and width of the box be?

Area = 96 in², length is 4 in. more than width

length = l
width = w
Area = l · w

Since the length is 4 inches more than the width, we can say, l = w + 4.

Area = l · w and l = w + 4, so

\[ \text{Area} = (w + 4) \cdot w \]

\[ 96 = (w + 4) \cdot w \]
\[ 96 = w^2 + 4w \]
\[ 0 = w^2 + 4w - 96 \]
\[ 0 = (w + 12)(w - 8) \]
\[ w = -12, 8 \]

Since we can’t have a negative distance, the answer is \( w = 8 \).

Since \( l = w + 4 \), \( l = 8 + 4 = 12 \).

**Answer: length = 12 inches, width = 8 inches**
The area of Janet’s triangular garden is $27 \, \text{ft}^2$. The base of the triangle is 3 feet less than the height. What are the dimensions of the garden?

Area = $27 \, \text{ft}^2$, base is 3 feet less than height
   base = $b$
   height = $h$

Area = $\frac{1}{2} \cdot b \cdot h$

Since the base is 3 feet less than the height, we can say, $b = h - 3$.

Area = $\frac{1}{2} \cdot (h - 3) \cdot h$ and $b = h - 3$, so

$\rightarrow 27 = \frac{1}{2} \cdot (h - 3) \cdot h$

$27 = \frac{1}{2} (h^2 - 3h)$

$27 = \frac{1}{2} h^2 - \frac{3}{2} h$

Clear the fractions by multiplying everything by 2: $54 = h^2 - 3h$

$0 = h^2 - 3h - 54$

$0 = (h - 9)(h + 6)$

$h = -6, 9$

Since we can’t have a negative distance, the answer is $h = 9$.

Since $b = h - 3$, then $b = 9 - 3 = 6$.

Answer: height = 9 feet, base = 6 feet
Factor the following.

1. $4x^2 + 36x$
2. $r^2 - 64$
3. $x^2 + 4x + 4$
4. $5x^2 - 4x - 1$
5. $x^2 - 25$
6. $2x^3 + 10x^2 - 3x - 15$
7. $-6 + 16a^2 + 20a$
8. $x^3 - 3x^2 + 2x$
9. $x^3 + 5x^2 - 2x - 10$
10. $x^2 + 16 + 63$
11. $25x^2 - 100x + 100$
12. $x^2 + 8x - 48$

The following are identical to the previous twelve problems, except that they are now equations. Now that you have factored them, solve for the variable.

13. $4x^2 + 36x = 0$
14. $r^2 - 64 = 0$
15. $x^2 + 4x + 4 = 0$
16. $5x^2 - 4x - 1 = 0$
17. $x^2 - 25 = 0$
18. $2x^3 + 10x^2 - 3x - 15 = 0$
19. $-6 + 16x^2 + 20x = 0$
20. $x^3 - 3x^2 + 2x = 0$
21. $x^3 + 5x^2 - 2x - 10 = 0$
22. $x^2 + 16 + 63 = 0$
23. $25x^2 - 80x + 64 = 0$
24. $x^2 + 8x - 48 = 0$

Factor the following and solve for the variable.

25. $4x^2 + 16x + 16 = 0$
26. $x^2 = 9$
27. $x^2 - 14x + 14 = -35$
28. $y^2 + 16 = 0$
29. $3x^2 - 36 = 3x$
30. $x^2 - 169 = 0$
31. $4x^2 + 36x - 15 = 25$
32. $x^3 + 3x^2 - 4x - 12 = 0$
33. $100x^2 + 80x + 16 = 0$
34. $x^2 - 4 = 0$
35. $6x^2 = -36x - 54$
36. $x^2 - x - 20 = 0$

Story Problems.

37. The energy of an object is dependent on its mass and can be described by the following equation: $E = 2m^2 - 12m$, where $E$ stands for energy and $m$ stands for mass. If the energy of the object is 14 units, what is the mass of the object?

38. The area of a window is 192 in$^2$. The width of the window is four inches more than half the length of the window. What are the dimensions of the window?

39. A cone has a surface area of $36\pi$ cm$^2$ and a slant height of 9 cm. What is the radius of the cone? (See section 2.2 for formulas)
### Section 5.6 Answers:

1. $4x(x + 9)$
2. $(r + 8)(r - 8)$
3. $(x + 2)^2$
4. $(5x + 1)(x - 1)$
5. $(x + 5)(x - 5)$
6. $(x + 5)(2x^2 - 3)$
7. $2(4a - 1)(2a + 3)$
8. $x(x - 1)(x - 2)$
9. $(x^2 - 2)(x + 5)$
10. $(x + 9)(x + 7)$
11. $25(x - 2)^2$
12. $(x + 12)(x - 4)$
13. $x = -9, 0$
14. $r = -8, 8$
15. $x = -2$
16. $x = -\frac{1}{5}, 1$
17. $x = -5, 5$
18. $x = -5, \sqrt{\frac{3}{2}} or \sqrt{\frac{6}{2}}$
19. $a = -\frac{3}{2}, \frac{1}{4}$
20. $x = 0, 1, 2$
21. $x = -5, \sqrt{2}$
22. $x = -9, -7$
23. $x = 2$
24. $x = -12, 4$
25. $x = -2$
26. $x = -3, 3$
27. $x = 7$
28. No solution
29. $x = -4, 3$
30. $x = -13, 13$
31. $x = -10, 1$
32. $x = -2, -3, 2$
33. $x = -2/5$
34. $x = -2, 2$
35. $x = -3$
36. $x = -4, 5$
37. $m = 7$
38. $l = 16\, \text{in}, w = 12\, \text{in}$
39. $r = 3\, \text{cm}$
The 5 Methods of Factoring

<table>
<thead>
<tr>
<th>Method</th>
<th>Type of Polynomial used on:</th>
</tr>
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<tr>
<td>1: Pull Out the Common Factor (C.F.)</td>
<td>All</td>
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<td>2: Grouping</td>
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<td>3: $ax^2 + bx + c$, where $a = 1$</td>
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5.1 Intro to Factoring (Methods 1 & 2)

**Factor:**
Numbers that can multiply together to get another number.

**Check Answers:**
You can always check your answer by multiplying to see if you get the original polynomial.

**Factoring Method 1:**
**Pull Out the Common Factor**

1. Identify the greatest common factor (C.F.) that ALL of the terms in the polynomial share in common.
2. Place the C.F. under each term in the polynomial.
3. Simplify each term that is now in the form: \(\frac{\text{Monomial}}{\text{C.F.}}\)
4. Write answer in this format: C.F. (Simplified Polynomial)

**Factoring Method 2:**
**Grouping**

1. Check for **method 1** first (Pull out C.F.)
2. Break the 4 terms into a (binomial) + (binomial)
3. Pull out the C.F. from **each** binomial to get C.F. (binomial) ± C.F. (binomial)
4. If the two (binomial)’s are identical, then they are now a common factor, so you can pull that out to get (binomial)(C.F. ± C.F.).

5.2 Factoring: Method 3

**Factoring Method 3:**
\(ax^2 + bx + c\) where $a = 1$

1. Check for **method 1** first (Pull out C.F.)
2. Place the trinomial in descending order or $x^2 + bx + c$.
3. Find factor pairs of $c$.
4. Find the two factors of $c$ that add up to $b$.
5. Answer format: \((x \pm factor)(x \pm factor)\) The $x$ is the square root of the first term. The two factors are from step 3, and their accompanying signs determine the ±.
5.3 Factoring: Method 4

**Factoring Method 4:**

\[ ax^2 + bx + c \quad \text{where} \; a \neq 1 \]

1. Check for method 1 first (Pull out C.F.)
2. Place the trinomial in descending order or \( ax^2 + bx + c \).
3. Multiply \( a \cdot c \).
4. Factor the product of \( a \cdot c \) to find the two factors that add to \( b \).
5. Break \( bx \) into two terms equal to the two factors found in step 4.
6. You now have four terms. Factor by grouping.

5.4 Factoring: Method 5 – Special Cases

**Factoring Method 5: Special Cases**

**Difference of Two Squares**

\[ (A^2 - B^2) = (A + B)(A - B) \]

1. Check for method 1 first (Pull out C.F.)
2. **CHECK:** Is this a binomial of the form \( (A^2 - B^2) \)? Are both terms perfect squares?
3. Take the square root of each term (don’t include signs)
4. Answer format: \( (A + B)(A - B) \)
   
   or \( (\sqrt{1\text{st term}} + \sqrt{2\text{nd term}})(\sqrt{1\text{st term}} - \sqrt{2\text{nd term}}) \)
5. Repeat these steps until no factoring is possible

**Factoring Method 5: Special Cases**

**Difference of Two Squares**

\[ (A + B)^2 = A^2 + 2AB + B^2 \]

1. Check for method 1 first (Pull out C.F.)
2. **CHECK:**
   - Is it a trinomial?
   - Are the first and last terms perfect squares?
   - Is the middle term \( 2 \times \sqrt{\text{first term}} \times \sqrt{\text{second term}} \) ?
   - Is the last term positive?
   - If the answer is YES to all of the above, continue. If not, try another method.
3. Take the square root of the first and last terms. \( \sqrt{\text{first term}} = A, \sqrt{\text{second term}} = B \)
4. Answer format: \( (A + B)^2 \) the sign is decided by the first sign in the trinomial
   
   or \( (\sqrt{\text{first term}} \pm \sqrt{\text{second term}})^2 \)
Steps to Factoring Polynomials

ALWAYS check for a common factor

Then look at the number of terms.

2 terms

Is the binomial is a difference of two squares?

yes

Factor: 

\[ A^2 - B^2 = (A + B)(A - B) \]

no

3 terms

Is it a perfect square trinomial?

yes

\[ A^2 \pm 2AB + B^2 = (A + B)^2 \]

no

Factoring:

\[ \alpha = 1 \]

Polynomial looks like:

\[ ax^2 + bx + c \]

4 terms

Factor by grouping

\[ \alpha \neq 1 \]

Polynomial looks like:

\[ ax^2 + bx + c \]

Check all of your factors using this chart to make sure you have factored completely.

You can ALWAYS check your answer by multiplying.
### Principle of Zero Products

If $a \cdot b = 0$, then either $a$ or $b$ or both are equal to **zero**.

### Solving Equations by Factoring

1. Arrange the equation so that it is equal to zero.
2. Factor the polynomial.
3. Set each factor equal to zero.
4. Solve each new equation for the variable.
5. Check your answer by substituting your answers into the original equation.

### Solving Story Problems

**D**- **Data.** Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

**V**- **Variable.** In all of these story problems, there is something that you don’t know, that you would like to. Pick any letter of the alphabet to represent this.

**P**- **Plan.** Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. Find a formula or draw a picture that helps you describe what is happening.

**E**- **Equation.** Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

*In quadratic equations, you usually get two answers. Only choose the answers that make logical sense, i.e. don’t choose a negative distance or a negative time.*
1. Create a visual chart of all of the methods, formulas, and examples from studying how to factor polynomials.

Identify the greatest common factor between the terms.

5.1

2. 112, 148, 246
3. $3j^2, 5j^2ay^3, 4j^3y$
4. $12m^4n^2p^4, 24m^3n^3p^4, 30m^3n^2p^5$

Factor the following by grouping.

5. $6x^3 + 10x^2 + 3x + 5$
6. $21x^3 - 14x^2 - 12x + 8$
7. $5ax^3 + 20a^2x^2 + 3x + 12a$
8. $28x + 7x^3 + 4 + x^2$

Factor the following using method 3 or method 4.

5.2-3

9. $2x^2 + 3x + 1$
10. $3x^2 - 12x - 63$
11. $2x^2 + 32x + 96$
12. $14x^2 + 29x - 15$
13. $-4x^2 - 24x + 108$
14. $7x^2 - 35x + 42$

Factor the following using method 5.

5.4

15. $-x^2 + 64$
16. $x^2 + 2x + 1$
17. $49x^2 - 14x + 4$
18. $x^4y^2 - 9z^2$
19. $-16x^2 - 4$
20. $8y^2 + 72y + 162$

Factor the following.

5.5

21. $18x^2 + 24x + 4$
22. $x^2 + 7x - 18$
23. $-18x^3 + 15x^2 + 24x - 20$
24. $-20x^2 - 19x - 3$
25. $-3a^2xy + 3b^2xy$
26. $x^2 - 15x + 56$
27. $x^2 - 23x + 132$
28. $x^2 - 4x - 45$
29. $14x^3 + 16x^2 + 35x + 40$
30. $-4x^2 + 52 - 168$
31. $24m^3 - 6m^2n - 63mn^2$
32. $49x^2 - 121$
33. $35x^4y^2 + 42x^4y + 15x^3y^2 + 18x^3y$
34. $4x^2 - 36xy + 81y^2$
Solve for the variable by factoring.

35. \( x^2 - 13x + 22 = -20 \)
36. \( -6x^2 - 11x + 10 = 0 \)
37. \( -5x^3 - 10x^2 = 5x \)
38. \( 8x^3 + 32x^2 + 2x + 8 = 0 \)
39. \( 100 - x^2 = -21 \)
40. \( 4x^3 + 4x^2 - 25x = 25 \)

Story Problems.

41. Henry shoots a rocket in his 10th grade science class. He found that the motion of the rocket can be described by the equation \( y = 144 - x^2 \), where \( y \) is the vertical distance and \( x \) is the horizontal distance. If this equation were to be graphed, find where the x-intercepts would be on the graph.

   *Fun Note: The positive answer to this question is actually the horizontal distance that the rocket flew from the peak of its path to the ground.

42. A triangle has an area of 14 square inches. The height of the triangle is three inches more than the height. What are the base and height of the triangle?

43. Jill has a small treasure box that is 6 inches long. It can hold a volume of 72 inches cubed, and the width of the box is 5 inches less than twice the height of the box. What are the dimensions of the box?
Chapter 5 Review 1

Answers

1. This is your last one, make it good
2. 2
3. $j^2$
4. $6m^3n^2p^4$
5. $(2x^2 + 1)(3x + 5)$
6. $(7x^2 - 4)(3x - 2)$
7. $(5ax^2 + 3)(x + 4a)$
8. $(x^2 + 4)(7x + 1)$
9. $(x + 1)(2x + 1)$
10. $3(x - 7)(x + 3)$
11. $2(x + 4)(x + 12)$
12. $(2x + 5)(7x - 3)$
13. $-4(x - 3)(x + 9)$
14. $7(x - 3)(x - 2)$
15. $(8 + x)(8 - x)$
16. $(x + 1)^2$
17. $(7x - 2)^2$
18. $(x^2y + 3z)(x^2y - 3z)$
19. $-4(4x^2 + 1)$, not a special case
20. $2(2y + 9)^2$
21. $2(3x + 2)^2$
22. $(x + 9)(x - 2)$
23. $(-3x^2 + 4)(6x - 5)$
24. $-1(5x + 1)(2x + 3)$
25. $-3xy(a + b)(a - b)$
26. $(x - 7)(x - 8)$
27. $(x - 11)(x - 12)$
28. $(x - 9)(x + 5)$
29. $(2x^2 + 5)(7x + 8)$
30. $-4(x - 7)(x - 6)$
31. $3m(8m^2 - 2mn - 21n)$
32. $(7x - 11)(7x + 11)$
33. $(7x^4y + 3x^3y)(5y + 6)$
34. $(2x - 9y)^2$
35. $x = 6, 7$
36. $x = -\frac{5}{2}, \frac{2}{3}$
37. $x = -1, 0$
38. $x = -4$
39. $x = -11, 11$
40. $x = -\frac{5}{2}, -1, \frac{5}{2}$
41. $(-12, 0), (12, 0)$
42. base = 4 inches, height = 7 inches
43. height = 4 inches, width = 3 inches
Identify the greatest common factor between the terms.

1. 52, 104, 182
2. $6j^2, 9j^2ay^3, 36j^4$
3. $54m^6n^3p^3, 72m^3n^8p^2, 162m^4np^3$

Factor the following by grouping.

4. $14a^3 + 10a^2 + 21a + 5$
5. $6xy - 3x + 2y + 1$
6. $40a^2b - 20ab + 40a + 15$
7. $7xy + 28x + y + 4$

Factor the following using method 3 or method 4.

8. $-8x^2 - 29x + 12$
9. $9 - m^2 + 6m$
10. $-28x^2 - 25x + 3$
11. $18x^2 + 36x - 1134$
12. $126a^2 - 83a - 143$
13. $18x^2 + 45x + 28$

Factor the following using method 5.

14. $144x^2 + 168x + 49$
15. $3p^2 - 363$
16. $16x^4 - 81$
17. $72x^3 - 8x$
18. $8x^2 - 24x + 18$
19. $1 - 9z^2$

Factor the following.

20. $8x^2 - 63x - 8$
21. $-14m + 6m^2$
22. $x^4 - 7x^2 + 12$
23. $12x^2 + 28xy - 3xy - 7y^2$
24. $-3m^4 + 9m^3 - 12m^2$
25. $25x^2 - 1$
26. $x^2 - 10x + 9$
27. $21z^2 + 7z^3 + 14z$
28. $9x^2 - 18x$
29. $25x^2 + 14x + 36$
30. $b^2 - 8b + 16$
31. $-x^2 + 4$
32. $10x^2 - 22x - 72$
33. $5x - 2x^2 - 2 + x^3$
Solve for the variable by factoring.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation</th>
</tr>
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<tbody>
<tr>
<td>34.</td>
<td>$0 = 3x^3 + x^2 - 4x$</td>
</tr>
<tr>
<td>35.</td>
<td>$9a^2 - 12a + 4 = 0$</td>
</tr>
<tr>
<td>36.</td>
<td>$12 - 7x^2 = x^4$</td>
</tr>
<tr>
<td>37.</td>
<td>$3x^2 + 4x = 0$</td>
</tr>
<tr>
<td>38.</td>
<td>$13x = 15x^2 + 2$</td>
</tr>
<tr>
<td>39.</td>
<td>$8 = 8x^2 - 63x$</td>
</tr>
</tbody>
</table>

Story Problems.

40. A company’s profit can be modeled by the equation $P = x^2 - 8x + 9$, where $P$ is the profit in thousands of dollars, and $x$ is the number of products sold in thousands. How many products would the company have to sell order to break even (a profit of zero)?

41. Kathy found a place online that sells picture frames that are twice as high as they are wide. She has a picture whose area is $128 \text{cm}^2$. What dimensions of picture frame should she order? This can be solved without factoring, by simply getting the variable by itself and simplifying. Solve that way and by factoring to see that they yield the same answer.

42. Jeffrey has a cylindrical oatmeal container with a surface area of $128\pi \text{ square inches}$. He wants to know the radius of the top part. He does know that the height is 12 inches. What is the radius? (See section 2.2 for formulas)
Answers
1. 2
2. \(j^2\)
3. \(6m^3n^2p^4\)
4. \((2x^2 + 1)(3x + 5)\)
5. \((7x^2 - 4)(3x - 2)\)
6. \((5ax^2 + 3)(x + 4a)\)
7. \((y + 4)(7x + 1)\)
8. \(-1(x + 4)(8x - 3)\)
9. \((m + 3)(m + 3)\ or\ (m + 3)^2\)
10. \(-(x + 1)(28x - 3)\)
11. \(18(x - 7)(x + 9)\)
12. \((9a - 13)(14a + 11)\)
13. \((6x + 7)(3x + 4)\)
14. \((12x + 7)^2\)
15. \(3(p + 11)(p - 11)\)
16. \((4x^2 + 9)(2x + 3)(2x - 3)\)
17. \(8x(3x + 1)(3x - 1)\)
18. \(2(2x - 3)^2\)
19. \((1 + 3z)(1 - 3z)\)
20. \((8x - 1)(x + 8)\)
21. \(2m(3m - 7)\)
22. \((x^2 - 3)(x^2 - 4)\)
23. \((4x - y)(3x + 7y)\)
24. \(-3m^2(m^2 - 3m + 4)\)
25. \((5x + 1)(5x - 1)\)
26. \((x - 9)(x - 1)\)
27. \(7z(z + 1)(z + 2)\)
28. \(9x(x - 2)\)
29. Prime
30. \((b - 4)^2\)
31. \((2 + x)(2 - x)\)
32. \(2(5x + 9)(x - 4)\)
33. \((x^2 + 5)(x - 2)\)
34. \(x = -\frac{4}{3}, 0, 1\)
35. \(a = \frac{2}{3}\)
36. \(x = -2, -\sqrt{3}, \sqrt{3}, 2\)
37. \(x = -\frac{4}{3}, 0\)
38. \(x = \frac{1}{2}\)
39. \(x = -8, \frac{1}{8}\)
40. \(x = 9;\ they\ have\ to\ sell\ 9,000\ products\)
41. \(8\text{cm} \times 16\text{cm}\)
42. radius = 4 inches