Exam 1 — Math 111 — Fall 2005
Bro. Woodruff — Sec. 302 — Nov 16 to Nov 17

Give the best answer to each question in the space provided. Where appropriate, put your answer in the box provided. Show your work. No calculators are allowed on this exam. Every problem is worth 4 points. Aside from evaluating trigonometric functions at multiples of 30 and 45 degrees, you may leave answers in unsimplified form.

1. Convert $\frac{5\pi}{12}$ radians to degrees.

$$\frac{5\pi}{12} \cdot \frac{180^\circ}{\pi} = 75^\circ$$

2. Find the radian measure of the least positive angle coterminal with $-\frac{19\pi}{4}$.

$$-\frac{19\pi}{4} + 3 \cdot 2\pi = \frac{5\pi}{4}$$

3. Determine whether the following are positive or negative
   (a) $\sin(-225^\circ)$

   positive

   (b) $\cos(-4\pi/3)$

   negative

   (c) $\tan(135^\circ)$

   negative

4. Find the exact value of $\frac{\sin(5\pi/6)}{1 + \cos(5\pi/6)}$.

$$\frac{1/2}{1 + (-\sqrt{3}/2)} = \frac{1}{2 - \sqrt{3}}$$
5. Find \( \cos(\alpha) \), given that \( \sin(\alpha) = -\frac{12}{13} \) and \( \alpha \) is in quadrant IV.

**Solution:** We know \( \sin^2(\alpha) + \cos^2(\alpha) = 1 \), hence \( \left( -\frac{12}{13} \right)^2 + \cos^2(\alpha) = 1 \).

So \( \cos^2(\alpha) = 1 - \frac{144}{169} = \frac{25}{169} \).

Hence \( \cos(\alpha) = \pm \frac{5}{13} \).

We chose the positive number because cosine is positive in quadrant IV.

\[
\frac{5}{13}
\]

6. Find the exact value of \( \tan\left(\frac{7\pi}{6}\right) \).

**Solution:** \( \tan\left(\frac{7\pi}{6}\right) = \frac{\sin\left(\frac{7\pi}{6}\right)}{\cos\left(\frac{7\pi}{6}\right)} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \). Either answer will be ok.

\[
\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\]

7. Find the exact value of \( \csc(-135^\circ) \).

**Solution:** \( \csc(-135^\circ) = \frac{1}{\sin(-135^\circ)} = \frac{1}{-\sqrt{2}/2} = -\frac{2}{\sqrt{2}} = -\sqrt{2} \). Either answer will be ok.

\[
-\frac{2}{\sqrt{2}} = -\sqrt{2}
\]

8. Compute \( \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) \).

**Solution:** The two angles closest to 0 whose sine is \( -\frac{\sqrt{2}}{2} \) are \( -\frac{\pi}{4} \) and \( \frac{3\pi}{4} \). So choose \( -\frac{\pi}{4} \).

\[
-\frac{\pi}{4}
\]

9. Compute \( \tan^{-1}\left(\sqrt{3}\right) \).

**Solution:** \( \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \sqrt{3} = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}/2}{1/2} \). Hence we pick \( \alpha = \pi/3 \). Or you can give this from memory.

\[
\pi/3
\]

10. Compute \( \arccos(-\sqrt{2}) \).

**Solution:** \( \arccos(-\sqrt{2}) = \arcsin(-1/\sqrt{2}) = \arcsin(-\sqrt{2}/2) = -\pi/4 \).

\[
-\pi/4
\]
11. Compute \( \arccos(\cos(-\pi/3)) \).

**Solution:** \( \arccos(\cos(-\pi/3)) = \arccos(1/2) = \pi/3. \)

\[ \pi/3 \]

12. Write an equation of the form \( A \sin(B[x - C]) + D \) whose graph is the given sine wave.

**Solution:** \( A = 3, P = 2, B = 2\pi/2 = \pi, C = 1/2, D = 6. \)

\[ y = 3 \sin(\pi [x - 1/2]) + 6 \]

13. The blade on a table saw rotates at 5280 revolutions per minute. What is the linear velocity in miles per hour of the outside edge of a 12-in. diameter blade? (Recall that there are 5280 ft in 1 mile) Simplify your answer.

**Solution:** \[ \frac{5280 \cdot 2\pi \cdot 60 \text{ mi}}{12 \cdot 5280 \text{ hr}} = 60\pi \frac{\text{mi}}{\text{hr}} \]

\[ 60\pi \frac{\text{mi}}{\text{hr}} \]
14. Solve the right triangle below, by labeling the unmarked edge and unmarked angles. Give exact values.

\[ \begin{array}{c}
6 \\
3 \\
\end{array} \]

Solution: I goofed and drew the 6 in the wrong spot. Give yourself 3 points for this problem, unless you got it completely correct. Sorry. The answers are
\[ \sqrt{45} \] for the unmarked edge,
and then the two angles are
\[ \sin^{-1}(3/\sqrt{45}) = \cos^{-1}(6/\sqrt{45}) = \tan^{-1}(3/6) \] for the bottom left angle, and
\[ \sin^{-1}(6/\sqrt{45}) = \cos^{-1}(3/\sqrt{45}) = \tan^{-1}(6/3) \] for the upper right.

15. For the past 3 years, Ben notices that his utility bill reaches a high of $90 in January and a low of $10 in July. In addition, a graph of his utility bill looks like a sinusoid. If the months are numbered 1 to 36, with 1 corresponding to January, write a formula to model Ben’s utility bill.

Solution: \( A = 40, B = 2\pi/12, C = 10 \) (or \(-2\)), \( D = 50 \).

\[ y = 40 \sin\left(\frac{\pi}{6}(x - 10)\right) + 50 \]

Alternatively you could have
\[ y = 40 \sin\left(\frac{\pi}{6}(x + 2)\right) + 50 \]
\[ y = 40 \cos\left(\frac{\pi}{6}(x - 1)\right) + 50 \]

16. Sketch at least one cycle of the graph of \( f(x) = 3 \cos(x + \pi/2) - 2 \). Determine the amplitude and phase shift.

Solution: The amplitude is 3, the period is \( 2\pi \), the phase shift is left \( \pi/2 \), and the vertical shift is down 2
17. Sketch at least one cycle of the graph of \( f(x) = \sin\left(\frac{\pi}{3}x\right) \).

**Solution:** The amplitude is 1, the period is 6, the phase shift is 0, and the vertical shift is 0.

18. Determine the period and sketch at least one cycle of the graph of \( y = \cot(2x) \).

**Solution:** The period is \( \pi/2 \).

19. Determine the period and sketch at least one cycle of the graph of \( y = 3\sec(\pi x) \).

**Solution:** The amplitude is 3, the period is 2, the phase shift is 0, and the vertical shift is 0.
20. Ben is hiking directly toward a long straight road when he encounters a pond. He turns 24° to the left and hikes 2 more miles before reaching the road. How far was Ben from the road before he made the turn? (Your answer may be left in terms of an unevaluated trigonometric function.)

Solution: We know that $\cos(24^\circ) = \frac{x}{2}$, where $x$ is the distance to the road. Solving for $x$ gives $x = 2\cos(24^\circ)$

21. A hot air balloon is between two spotters who are 2 miles apart. One observer reports that the angle of elevation to the balloon is 45°, and the other reports that the angle of elevation is 30°. Assuming that the ground between the two observers is level, how high is the balloon? Your answer should be given in terms of square roots.

Solution: Begin by labeling the height $h$ and then the distances on the bottom are $h$ and $x$, as shown in the diagram. We know the two $h$ distances are equal because of the 45° angle. We now have

$h + x = 2$ and $\tan(30^\circ) = \frac{h}{x}$, or $\frac{\sqrt{3}}{3} = \frac{h}{x}$, or $\sqrt{3}h = x$.

We hence have $h + \sqrt{3}h = 2$, so $h(1 + \sqrt{3}) = 2$,

which becomes $h = \frac{2}{1 + \sqrt{3}}$. 

\[
\frac{2}{1 + \sqrt{3}}
\]