Applications of Trigonometry

Math 111 — Trigonometry

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Law of Sines and Law of Cosines
Labels

- Lower case letters for side lengths: $a, b, c$.
- Greek letters for angles: $\alpha, \beta, \gamma$.
- Angles match opposite side.
Solving Oblique Triangles

- To solve a triangle you need 3 parts. One must be a length.
  1. Two angles, one side (AAS or ASA).
  2. Two sides, one angle opposite the given side (SSA).
  3. Two sides, the angle inbetween (SAS).
  4. Three sides (SSS).

- Law of sines solves AAS, ASA, or SSA.
  \[
  \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
  \]

- Law of cosines solves SAS or SSS.
  \[
  c^2 = a^2 + b^2 - 2ab \cos \gamma
  \]

- All could be solved by drawing right triangles.
Law of Sines

- The law of sines says that ratio of the sine of an angle and its opposite edge is the same for all angles.

\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
\]

- The area \( A \) of the triangle is

\[
A = \frac{1}{2} (\text{base})(\text{height}) = \frac{1}{2} bc \sin \alpha = \frac{1}{2} ac \sin \beta = \frac{1}{2} ab \sin \gamma
\]
Law of Cosines

- The law of cosines is similar to the Pythagorean theorem:

\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]
\[ b^2 = a^2 + c^2 - 2ac \cos \beta \]
\[ a^2 = b^2 + c^2 - 2bc \cos \alpha \]

- Subtract from the square of the other sides, twice their product times the cosine of the opposite angle.

- Heron’s Area Formula:

\[ A = \sqrt{S(S - a)(S - b)(S - c)} \quad \text{where} \quad S = \frac{a + b + c}{2} \]
AAS, ASA

- **Angle-Angle-Side (AAS):** We know \( \alpha, \beta, a \).
  
  \[
  \gamma = 180 - \alpha - \beta \\
  \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
  \]

- **Angle-Side-Angle (ASA):** We know \( \alpha, c, \beta \).
  
  \[
  \gamma = 180 - \alpha - \beta \\
  \frac{\sin \gamma}{c} = \frac{\sin \alpha}{a} = \frac{\sin \beta}{b}
  \]

- We must have \( \alpha + \beta < 180 \).
SSA

- Side-Side-Angle (SSA): We know $\alpha, a, b$.
- There are multiple cases.
- Construct the
Side-Angle-Side (SAS): We know $\alpha, b, c$. 
Side-Side-Side (SSS): We know $a, b, c$. 
Bearings
Extra Examples
Extra Examples II
Vectors
Extra Examples
Complex Numbers
Examples

1. Find \((2 + i)(3 - i)\)

2. Find \(\frac{1 + 2i}{4 - i}\)

3. Find \([3(\cos 20 + i \sin 20)][2(\cos 10 + i \sin 10)]\)

4. Find \([2(\cos -8 + i \sin -8)][7(\cos 53 + i \sin 53)]\)

5. Find \(\frac{6(\cos 8 + i \sin 8)}{2(\cos 53 + i \sin 53)}\)

6. Find \(\frac{3(\cos 45 + i \sin 45)}{2(\cos -45 + i \sin -45)}\)

7. Find \([2(\cos 30 + i \sin 30)]^4\)

8. Find \([8(\cos 135 + i \sin 135)]^{1/3}\)

9. Find \([1(\cos 270 + i \sin 270)]^{1/6}\)

10. Solve \(x^6 + 1 = 0\)
1. Find \((4 + i)(2 - 3i)\)

2. Find \(\frac{2 + i}{3 + i}\)

3. Find \([4(\cos 80 + i \sin 80)][3(\cos 70 + i \sin 70)]\)

4. Find \([2(\cos 12 + i \sin 12)][(\cos 78 + i \sin 78)]\)

5. Find \(\frac{9(\cos 27 + i \sin 27)}{3(\cos 27 + i \sin 27)}\)

6. Find \(\frac{2(\cos -45 + i \sin -45)}{10(\cos 135 + i \sin 135)}\)

7. Find \([1(\cos 40 + i \sin 40)]^3\)

8. Find \([16(\cos 180 + i \sin 180)]^{1/4}\)

9. Find \((2 - 2i)^{1/3}\)

10. Solve \(x^4 + 1 = 0\)
Polar Equations
Examples

1. Convert \((2, \pi/4)\) from polar to rectangular coordinates. Plot the point.

2. Convert \((-1, 1)\) from rectangular to polar coordinates.

3. Graph \(r = \cos \theta\). Plot at least 4 points. Convert to rectangular.

4. Graph \(r = 2 \sin \theta\). Plot at least 4 points. Convert to rectangular.

5. Graph \(r = \cos 3\theta\). Plot at least 4 points. Convert to rectangular.

6. Graph \(r = \sin 2\theta\). Plot at least 4 points. Convert to rectangular.

7. Convert \(y^2 = x\) to polar coordinates.

8. Convert \(y = x\) to polar coordinates.
Extra Examples
Parametric Equations
1. Plot the parametric curve given by \( x = 2t - 1, y = 3t + 2 \) for \( 0 \leq t \leq 3 \).
2. Graph the curve given by \( x = \cos t, y = \sin t \) for \( 0 \leq t \leq 2\pi \).
3. Graph and eliminate the parameter for
   \[ x = -t + 1, \quad y = 2t - 1. \]
4. Graph and eliminate the parameter for \( x = \sin t, y = \sin t \).
5. Write a pair of parametric equations that gives a line that starts at \((1, 0)\) for \( t = 0 \), and ends at \((3, -4)\) for \( t = 2 \).
6. Write a pair of parametric equations that gives a line that starts at \((-10, 5)\) for \( t = 0 \), and ends at \((8, 1)\) for \( t = 4 \).