QUANTITATIVE REASONING

Daily Life

Quantitative Skills

College Course Work  Career Experiences
Question: “What is the most important thing a person can do to improve his or her critical thinking skills?”

Phyllis Evitch, Rice Lake, Wisconsin

Answer: “Study logic. Without a sound foundation in the principles of reasoning you’ll be less able to understand your world, and the ramifications of this will ripple through everything from work to play. Even worse, you won’t realize what you’re missing.”

Marilyn Vos Savant
EXAMPLE ARGUMENT:

Bill is really smart, and all smart people are nerds. So, Bill must be a nerd.

PREMISE: Bill is really smart.
PREMISE: All smart people are nerds.
CONCLUSION: Bill is a nerd.
FALLACY: Appeal to Popularity

"So many people think they have seen Bigfoot. He must be real."

"I read the other day that most people really like the new gun control laws. I was sort of suspicious of them, but I guess if most people like them, then they must be okay."

FALLACY: False Cause

“After my opponent took office, the economy plummeted. A vote for me is a vote for restoring the economic engine of this country.”

“I got a date with Tom the day after my braces came off. Having my braces off is giving a boost to my dating life.”

FALLACY: Appeal to Ignorance

“Despite thousands of so called ‘sightings,’ no hard evidence for UFO's has ever been produced. Therefore UFO's don't exist.”

“You can’t prove that God exists, so he doesn’t.”

FALLACY: Hasty Generalization

“Two 747's were involved in air disasters in the past year. It must not be a safe plane.”

“I've never done well in my math classes. So, I’m sure I won’t do well in this one either.”
FALLACY: Limited Choice

"If you’re so smart, why aren’t you rich?"

If you don’t vote for me, you are voting against America.

“Have you stopped beating your wife?”

FALLACY: Appeal to Emotion

“The new UltraSkinny diet will make you feel great. No longer be troubled by your weight. Enjoy the admiring stares of the opposite sex. Revel in your new freedom from fat. You will know true happiness if you try our diet!”

FALLACY: Personal Attack

"Jane says that drug use is morally wrong, but she is just a goody-two shoes Christian, so we don't have to listen to her."

“Why shouldn't I hunt animals if I want? You eat meat, don't you?”

FALLACY: Circular Reasoning

"If such actions were not illegal, then they would not be prohibited by the law."

“Freedom of speech is an essential right in a free society, since everyone should have the right to express him or herself with complete freedom.”
FALLACY: Red Herring (Diversion)

"I think there is great merit in making the requirements stricter for the graduate students. I recommend that you support it, too. After all, we are in a budget crisis and we do not want our salaries affected."

FALLACY: Straw Man

"Senator Jones says that we should not fund the attack submarine program. I disagree entirely. I can't understand why he wants to leave us defenseless like that."
Prof. Jones: "The university just cut our yearly budget by $10,000."
Prof. Smith: "What are we going to do?"
Prof. Brown: "I think we should eliminate one of the teaching assistant positions. That would take care of it."
Prof. Jones: "We could reduce our scheduled raises instead."
Prof. Brown: "I can't understand why you want to bleed us dry like that, Jones."

"On our watch, the crime rate has gone down. Obviously, our policies are succeeding in the war against crime."

"The Book of Mormon can't possibly be true. Despite what those Mormons say, no real evidence has been found of Book of Mormon people having lived in the Americas."

"My fellow Americans...there has been some talk that the government is overstepping its bounds by allowing police to enter peoples' homes without the warrants traditionally required by the Constitution. However, these are dangerous times and dangerous times require appropriate actions. I have in my office thousands of letters from people who let me know, in no uncertain terms, that they heartily endorse the war against crime in these United States. Because of this overwhelming approval, it is evident that the police are doing the right thing."

"Someone who would mutilate and kill a little child deserves to be killed – I'd throw the switch myself with a smile."

"You know, I've begun to think that there is some merit in the Republican's tax cut plan. I suggest that you come up with something like it, because if we Democrats are going to survive as a party, we have got to show that we are as tough-minded as the Republicans, since that is what the public wants."

Bill: "Jill and I both support having prayer in public schools."
Jill: "Hey, I never said that!"
Bill: "You're not an atheist are you Jill?"
"Everyone in a wealthy society like ours has a right to health care, therefore health care should be made universally available."

Bill and Jill are arguing about cleaning out their closets:
Jill: "We should clean out the closets. They are getting a bit messy."
Bill: "Why, we just went through those closets last year. Do we have to clean them out everyday?"
Jill: "I never said anything about cleaning them out every day. You just want too keep all your junk forever, which is just ridiculous."

“Does your procrastination cause your grades to suffer?”

Bill: "I think that some people have psychic powers."
Jill: "Do you have any proof or evidence to support your belief?"
Bill: "Well, no one has been able to prove that people do not have psychic powers."

Bill: "God must exist."
Jill: "How do you know."
Bill: "Because the Bible says so."
Jill: "Why should I believe the Bible?"
Bill: "Because the Bible was written by God."

Sam is riding her bike in her home town in Idaho, minding her own business. A station wagon comes up behind her and the driver starts beeping his horn and then tries to force her off the road. As he goes by, the driver yells "get on the sidewalk where you belong!" Sam tells her roommates about the incident, concluding that all Utah drivers are jerks.

Bill: "I don't think it is a good idea to cut social programs."
Jill: "Why not?"
Bill: "Well, many people do not get a fair start in life and hence need some help. After all, some people have wealthy parents and have it fairly easy. Others are born into poverty and..."
Jill: "You just say that stuff because you have a soft heart and an equally soft head."
“A good and cheerful attitude is key to staying healthy. After all, people who are severely ill are very often depressed and angry. So the depression and anger must be the cause of the severe illness.”

Bill: "You know, those feminists all hate men."
Joe: "Really?"
Bill: "Yeah. I was in my philosophy class the other day and that Rachel chick gave a presentation."
Joe: "Which Rachel?"
Bill: "You know her. She's the one that runs that feminist group over at the Women's Center. She said that men are all sexist pigs. I asked her why she believed this and she said that her last few boyfriends were real sexist pigs."
Joe: "That doesn't sound like a good reason to believe that all of us are pigs."
Bill: "That was what I said."
Joe: "What did she say?"
Bill: "She said that she had seen enough of men to know we are all pigs. She obviously hates all men."
Joe: "So you think all feminists are like her?"
Bill: "Sure. They all hate men."
Which of the following are considered propositions?

1. Cows can fly.
2. Sister Youngberg likes cheesecake.
3. Come back tomorrow.
4. \( \frac{1}{2} = \frac{2}{3} \)
5. What is your favorite color?
6. \( 3x^2 - 2x + 1 \)
7. Some students will ace the quiz next class period.
FIND THE NEGATION
OF EACH PROPOSITION

1. Nobody likes ice cream.

2. Miracles happen.

3. All politicians are liars.

4. Andrew Wiles proved Fermat’s Last Theorem.

5. Some of my friends are afraid of taking risks.


7. Caviar isn’t one of my favorites.

8. Some math 108 students aren’t paying attention.

9. All dogs go to heaven.

10. Somebody is going to pay for this.
“I cannot say that I do not disagree with you.”

-Groucho Marx

“I don’t want no sympathy from you.”

“He ain’t got no talent.”
"If we pull this off, we'll eat like kings."
Suppose your basketball coach made the following if-then statement:

“If you play well in practice, then you will start in tomorrow’s game.”

In which of the following cases would you feel that you had been treated unfairly and that the coach lied to you?

- You play well in practice (antecedent true)
  You start the game (consequent true)

- You play well in practice (antecedent true)
  You do not start the game (consequent false)

- You do not play well in practice (antecedent false)
  You start the game (consequent true)

- You do not play well in practice (antecedent false)
  You do not start the game (consequent false)

A conditional statement is only false when the **antecedent** is true and the **consequent** is false.
Rephrase each of the following conditional propositions so that it is in the standard if…then… form:

1. All dogs go to heaven.

2. He’s over 35 if he’s president of the United States.

3. Taking Math 108 is sufficient to meet the general education math requirement.

4. Every square is a rectangle.

5. Babies cry whenever they are hungry.

6. No turkeys like Thanksgiving.
TRUE OR FALSE?

1. George Bush or Elvis Presley was a U.S. president.
2. $1+1=2$ and $2+3=4$.
3. Cats and lizards are mammals.
4. Oregon is a U.S. state or Idaho is a U.S. state.
5. Some people are tall and some people are short.

6. If cows can fly, then pigs can fly.
7. If cows can fly, then sparrows can fly.
8. If horses are mammals, then lizards are mammals.
9. If a student takes MATH 110, he/she has met the general ed. math requirement.
10. If a person is running, then he/she is breathing.
Are the following two statements logically equivalent?

\[ \sim(p \lor q) \]
\[ \sim p \lor \sim q \]

(Use a truth table to decide.)

To reinforce your conclusion from above, compare the statements below. Which two statements have essentially the same meaning?

1. I’m not hungry or tired.
2. I’m not hungry or I’m not tired.
3. I’m not hungry and I’m not tired.
MORE PRACTICE WITH LOGICAL CONNECTORS AND TRUTH TABLES:

1. Determine whether the following statements are logically equivalent:

   \[ \sim ( q \Rightarrow p ) \]

   \[ q \& \sim p \]

2. Determine whether the following statements are logically equivalent:

   \[ \sim ( q \Rightarrow p ) \]

   \[ \sim q \& p \]

3. Suppose that \( p \) is a false proposition, and \( q \) and \( r \) are true propositions. Determine the truth value of the following statements:

   \[ \sim p \& ( q \Rightarrow p ) \]

   \[ (\sim q \text{ or } r) \& \sim p \]
VALID CONDITIONAL ARGUMENT FORMS

AFFIRMING THE ANTECEDENT

EXAMPLE: If you study, then you’ll pass. Meg studies. Therefore, Meg will pass.

FORM: \[ p \Rightarrow q \]
       \[
       \begin{array}{c}
       p \\
       q
       \end{array}
       \]

DENYING THE CONSEQUENT

EXAMPLE: If you study, then you’ll pass. Mark didn’t pass. He must not have studied.

FORM: \[ p \Rightarrow q \]
       \[
       \begin{array}{c}
       q \\
       \sim q \\
       \sim p
       \end{array}
       \]
INVALID CONDITIONAL ARGUMENT FORMS
(I.E. FALLACIES)

ASSUMING THE INVERSE
(DENYING THE ANTECEDENT)

EXAMPLE: If you study, then you’ll pass. Melissa didn’t study. Melissa won’t pass.

FORM: \[ \text{p} \implies \text{q} \]
\[ \neg \text{p} \]
\[ \neg \text{q} \]

ASSUMING THE CONVERSE
(AFFIRMING THE CONSEQUENT)

EXAMPLE: If you study, then you’ll pass. Melissa passed, so Melissa must have studied.

FORM: \[ \text{p} \implies \text{q} \]
\[ \text{q} \]
\[ \text{p} \]
If you study a lot, then you will get good grades. Josh received good grades. Therefore, Josh studied a lot.

All true friends are trustworthy. Jason is not trustworthy. Therefore, Jason is not a true friend.

If Mark fails the final, then he will fail the course. Mark did not fail the final. Therefore, Mark will not fail the course.

If we don’t have it then you don’t need it. You need it. Therefore, we have it.

If it rains on Tuesday, then it will be sunny on Wednesday. It was sunny on Wednesday. Therefore, it rained on Tuesday.
INTERPRETING A SURVEY

A survey asked newspaper readers which of the following three newspapers they read daily:

- New York Times (NYT)
- Washington Post (WP)
- Wall Street Journal (WSJ)

The results are shown in the following table:

<table>
<thead>
<tr>
<th>PAPER(S)</th>
<th>NUMBER OF READERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYT only</td>
<td>24</td>
</tr>
<tr>
<td>WSJ only</td>
<td>27</td>
</tr>
<tr>
<td>WP only</td>
<td>26</td>
</tr>
<tr>
<td>NYT and WSJ only</td>
<td>14</td>
</tr>
<tr>
<td>NYT and WP only</td>
<td>16</td>
</tr>
<tr>
<td>WP and WSJ only</td>
<td>13</td>
</tr>
<tr>
<td>all three</td>
<td>8</td>
</tr>
<tr>
<td>none</td>
<td>15</td>
</tr>
</tbody>
</table>

Draw a Venn diagram representing the data in the table, and answer the following questions:

1. How many people surveyed read the New York Times?
2. How many people read the Washington Post and the Wall Street Journal?
3. How many people read the New York Times or Washington Post?
4. How many people read at least one of the three papers mentioned in the survey?
5. What percentage of the people surveyed read the Wall Street Journal?
In a group of 100 people, 10 are taking neither English nor math. 75 are taking English and 65 are taking math. How many are taking both English and math?

In a group of 37 people, 18 are neither overweight nor lawyers. Ten are overweight and 13 are lawyers. How many lawyers in the group are not overweight?

In a survey, 60% of those surveyed owned a car, and 80% of those surveyed owned a TV. If 55% owned both a car and a TV, what percent of those surveyed owned a car or a TV but not both?
Water boils down to nothing... snow boils down to nothing... ice boils down to nothing...
ARGUMENTS INVOLVING CATEGORICAL PROPOSITIONS

All salty foods cause high blood pressure. Apples do not cause high blood pressure. So apples are not salty.

All blondes are gullible. Mary is gullible, so Mary must be blonde.

No criminals are happy. I am not a criminal, so I am happy.

No swimmers are afraid of water. Melissa is afraid of water. Melissa must not be a swimmer.

All cheerleaders are outgoing. Dave is a cheerleader, so he must be outgoing.
ARGUMENTS INVOLVING A CHAIN OF CONDITIONALS

If I get sick, then I will miss school. If I miss school, then I will get behind in my classes. So, if I get sick, then I will get behind in my classes.

If you score at least a 90%, then you’ll earn an A. If you earn an A, then your parents will be proud. Thus, if your parents are proud, you scored a 90%.

If I do not eat, then I will sleep. I will not eat if I am worried. Hence, if I am worried, I will sleep.

If I do not eat, then I will sleep. I will not eat if I am worried. Hence, if I sleep, I am worried.
Use a Venn diagram to decide whether the following argument is valid:

PREMISE: If you read *The Firm* by John Grisham, you’ll become a fan of John Grisham’s books.

PREMISE: No Grisham fans like movies made from his books.

PREMISE: Joe likes movies made from Grisham books.

CONCLUSION: Joe must not have read *The Firm*. 
<table>
<thead>
<tr>
<th><strong>INDUCTIVE ARGUMENT</strong></th>
<th><strong>DEDUCTIVE ARGUMENT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>specific examples</td>
<td>general premises</td>
</tr>
<tr>
<td>general conclusion</td>
<td>more specific conclusion</td>
</tr>
<tr>
<td><strong>cannot</strong> be used to <strong>prove</strong> its conclusion</td>
<td><strong>can</strong> be used to <strong>prove</strong> its conclusion</td>
</tr>
<tr>
<td>evaluated in terms of <strong>strength</strong></td>
<td>evaluated in terms of <strong>validity</strong> and <strong>soundness</strong></td>
</tr>
</tbody>
</table>
INDUCTIVE VS. DEDUCTIVE

Decide whether each of the given arguments is inductive or deductive.

1. I’ve liked every Ben Affleck movie that I’ve ever seen, so I’m sure that I’ll like his new one.

2. When there’s a bad accident on the freeway, the police block off a lane. If a lane is blocked off, we’ll be late to work. There was a bad accident on the freeway this morning, so we’ll be late to work today.

3. I like working out at any gym that has a swimming pool. Jack’s gym has a pool, so I know I’d enjoy working out there.

4. I like Chinese cuisine and I like Thai cuisine, so I’d also like any other Asian cuisine.
Use inductive reasoning to decide whether the following statements are true mathematical rules:

1. The difference of two positive numbers is always positive.

2. For all positive integers, $n$,

   $$1 + 2 + 3 + \ldots + n = \frac{n}{2}(n+1)$$

3. For all numbers $a$ and $b$,

   $$(a + b)^2 = a^2 + b^2$$
Consider the following algebraic expression: \( n^2 - n + 11 \)

It appears that
\[ n^2 - n + 11 \]
will always equal a prime number when \( n \geq 0 \).

Or does it?

How about \( n = 11 \)?

\[ 11^2 - 11 + 11 = 121 \]

(a non-prime counterexample)